UNIVERSIDADE FEDERAL DO CEARÁ

UFOP, Departamento de Engenharia Civil
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PROPEC PROGRAMA DE PÓS GRADUAÇÃO EM ENGENHARIA CIVIL

Nonlinear Multi-Scale Finite Element Modeling of the Progressive Collapse of Reinforced Concrete Structures

Lecture 2: Newton-Raphson (recall)

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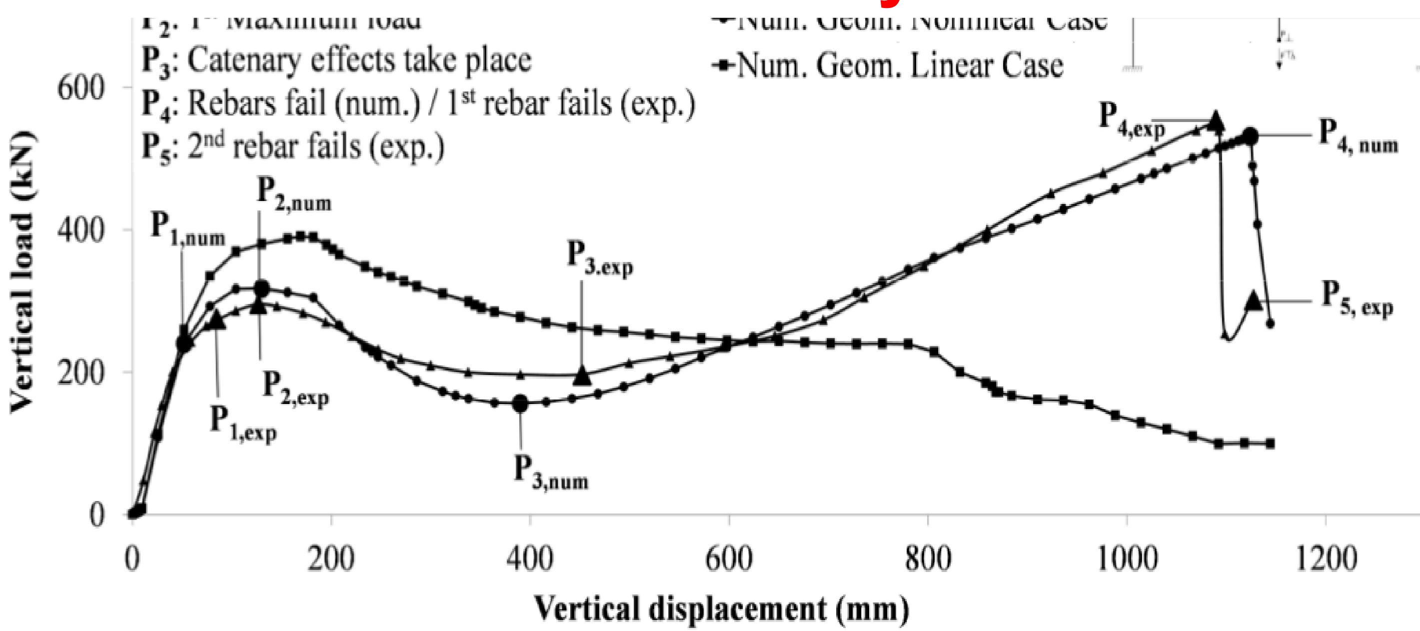
Example of nonlinear structural response

[3]F. Sadek et al., *J. Struct. Engng. ASCE*, **137**, 881-892 (2011)

[4]H.S. Lew et al., NIST Technical Note 1720 (2011)

How to solve a nonlinear problem numerically?



Outline

Newton-Raphson procedure for 1 NL equation

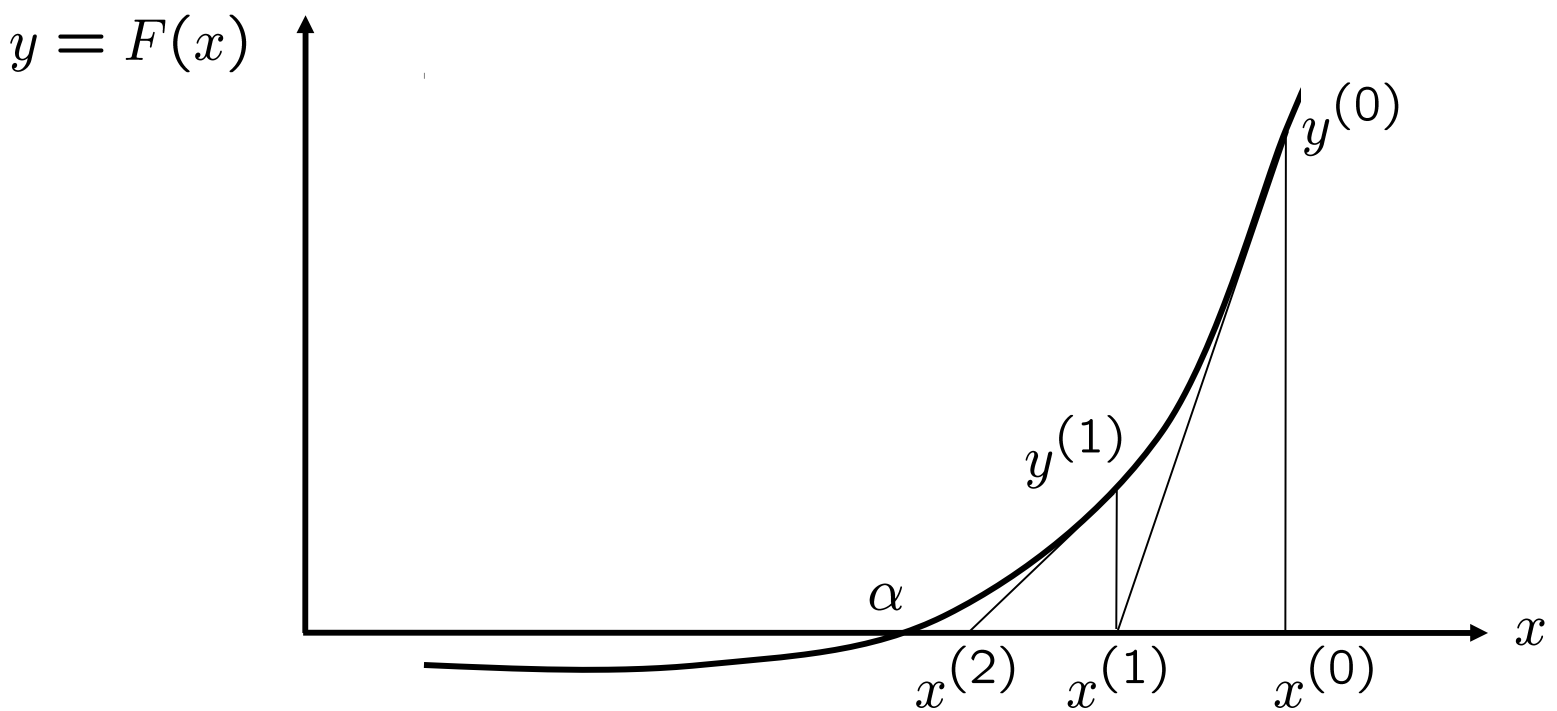
N-R in the structure of a NL FE code

Lab: Solve 1 NL equation using N-R in MatLab

Newton-Raphson procedure

Problem statement

Find a new approximation based on an initial value and the slope of the function at this point



Newton-Raphson procedure

Iterative scheme

Re-write $F(x) = 0$ under the form $x = f(x)$

Construct a series of successive approximations

$$x^{(1)} = f(x^{(0)})$$

$$x^{(k)} = f(x^{(k-1)})$$

Newton-Raphson approximation:

$$x^{(1)} = x^{(0)} - \frac{F(x^{(0)})}{F'(x^{(0)})}$$

$$x^{(k+1)} = x^{(k)} - \frac{F(x^{(k)})}{F'(x^{(k)})}$$

- Requires the knowledge of the function's derivative
- Quadratic <u>local</u> convergence, <u>if the derivative is right</u>
- This last point is CRUCIAL for a proper convergence!

Newton-Raphson procedure

Interpretation from a series development

Assume a first approximation is available $x^{(k)}$

Express the value of the function as a first order development

$$F(x^{(k+1)}) = F(x^{(k)}) + F'(x^{(k)})(x^{(k+1)} - x^{(k)}) + \frac{F''(x^{(k)})}{2!}(x^{(k+1)} - x^{(k)})^2 + \ldots$$

If this new value has to vanish (to find the root)

$$F(x^{(k+1)}) = 0$$

Using the first order development, a new approximation is

$$F(x^{(k)}) + F'(x^{(k)})(x^{(k+1)} - x^{(k)}) \approx 0$$

$$x^{(k+1)} = x^{(k)} - \frac{F(x^{(k)})}{F'(x^{(k)})}$$

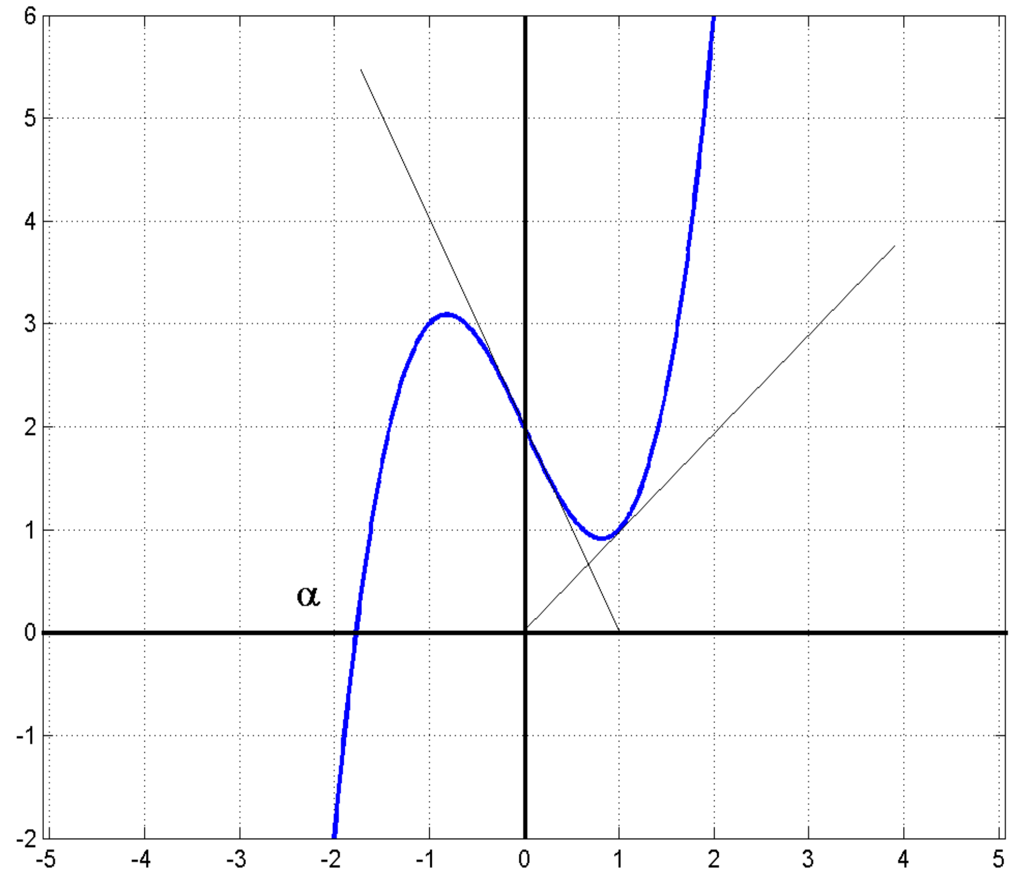
Newton-Raphson procedure

Potential shortcomings

Vanishing derivatives

Deadlocks between particular points

$$F(x) = x^3 - 2x + 2$$



Newton-Raphson procedure

System of equations

$$F_1(x_1, ..., x_n) = 0$$
$$\cdots$$
$$F_n(x_1, ..., x_n) = 0$$

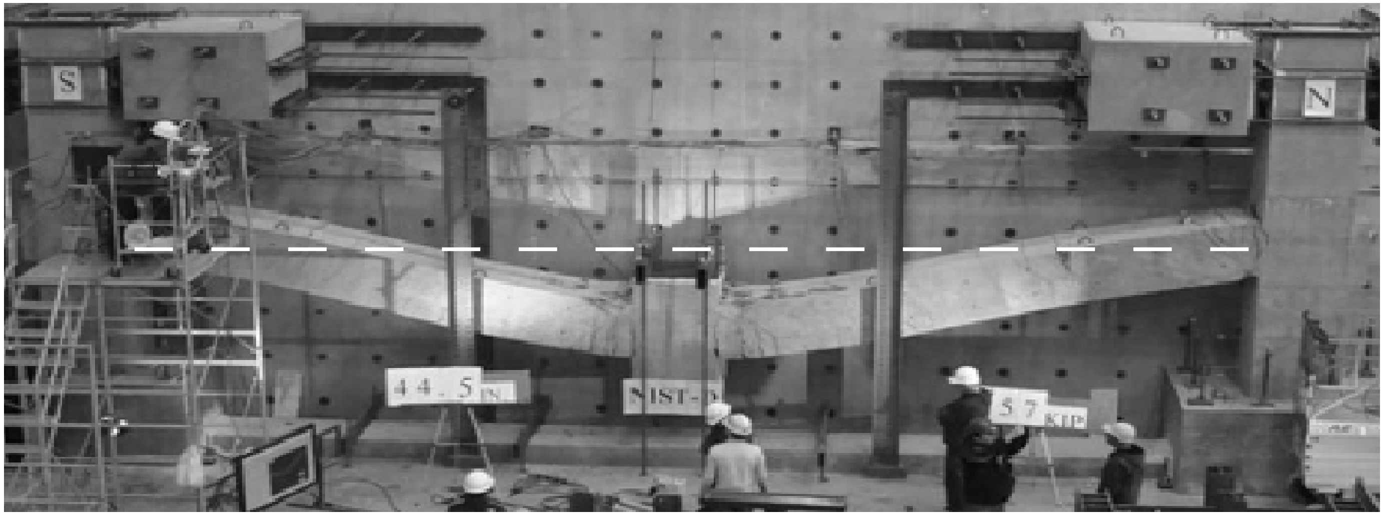
Initial approximation: $\left\{x^{(0)}\right\} = \left\{x_1^{(0)}, ..., x_n^{(0)}\right\}^T$

A new approximation is found by solving:

$$\left[J_F\left(\left\{x^{(k)}\right\}\right)\right]\left(\left\{x^{(k+1)}\right\} - \left\{x^{(k)}\right\}\right) = -\left\{F\left(\left\{x^{(k)}\right\}\right)\right\}$$

Jacobian matrix

Structure of a NL FE code



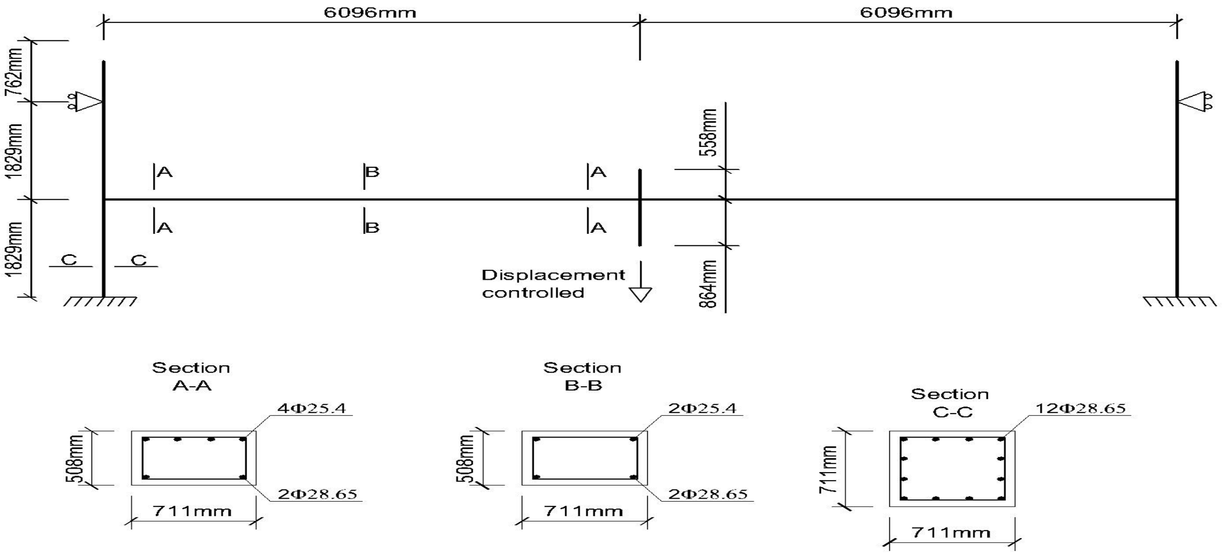
Objectives:

- Determine the load-displacement response of a structure
- Determine some of its points (limit load, …)
- Determine the state of the structure for various load levels

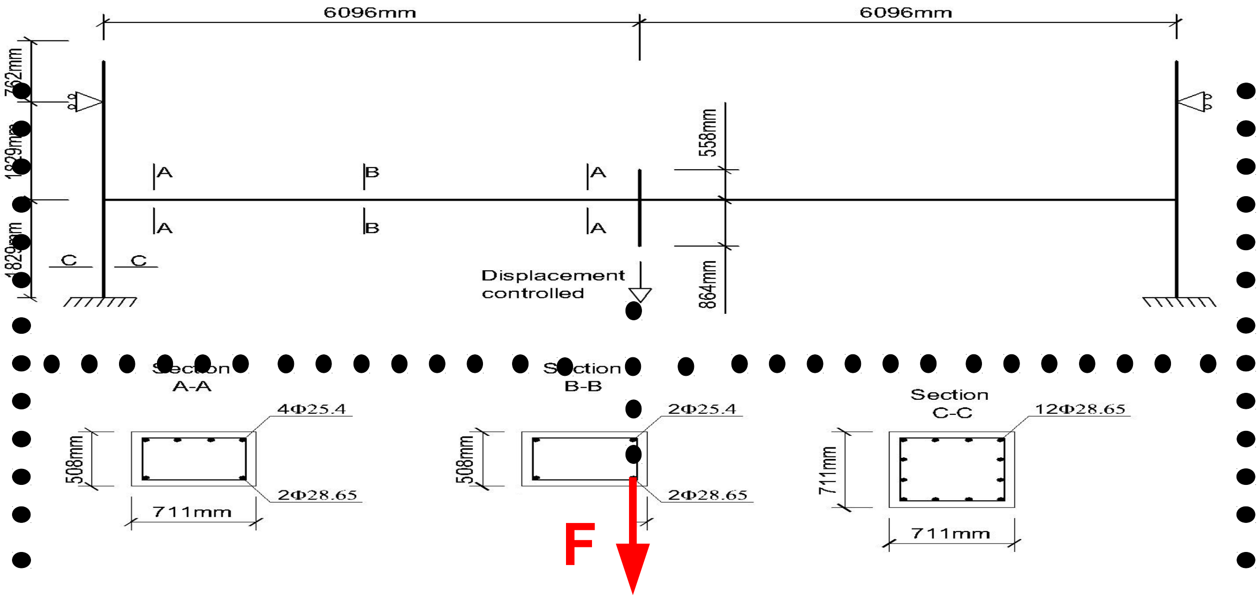
Computational method:

- Local convergence; loads have to be applied step-by-step
- Series of successive problems solved in an iterative fashion

Structure of a NL FE code



Structure of a NL FE code



Searching for structural equilibrium

Structure of a NL FE code

Flowchart

Define a set of successive loading states $F_{ext,n}$

Loop on the loading states (steps or increments)

Formulate the problem for the step $n \to n+1$

Find q_{n+1} such that $F_{int}(q_{n+1}) - F_{ext,n+1} = 0$

With as first approximation $q_{n+1}^{(0)} = q_n$

Iterate until a precision threshold is reached with

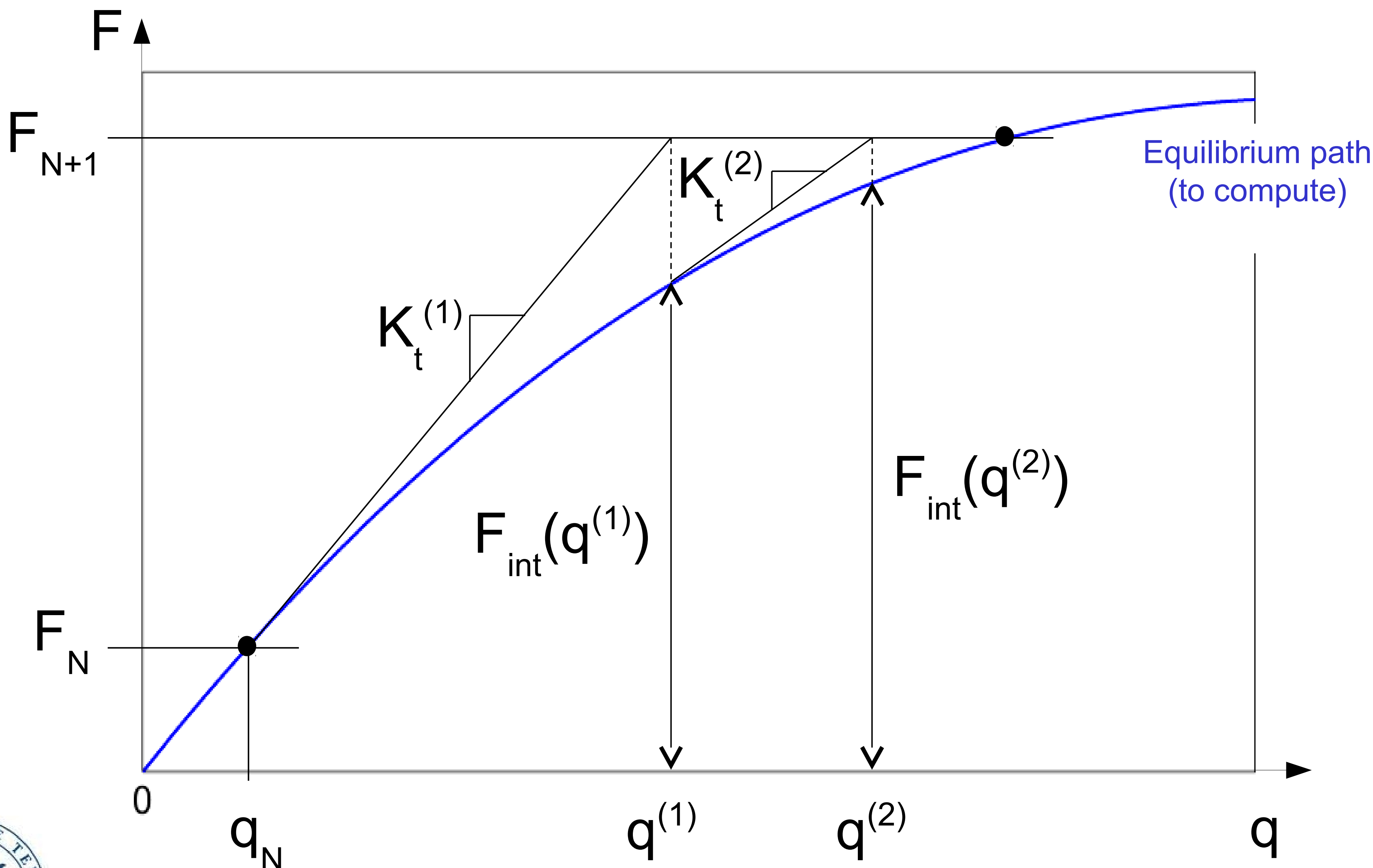
$$F_{int}^{(k)} = F_{int}\left(q_{n+1}^{(k)}\right)$$

$$q_{n+1}^{(k+1)} = q_{n+1}^{(k)} - \underbrace{\left(\frac{\partial F_{int}}{\partial q}\right)_{q_{n+1}^{(k)}}^{-1}}_{K_T} \left(F_{ext,n+1} - F_{int,n}^{(k)}\right)$$

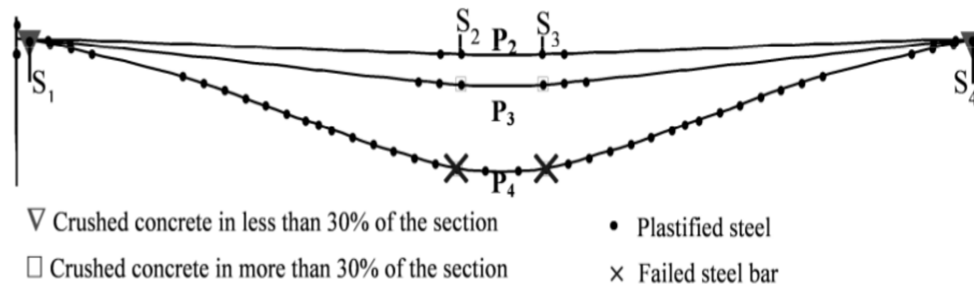
End of increment

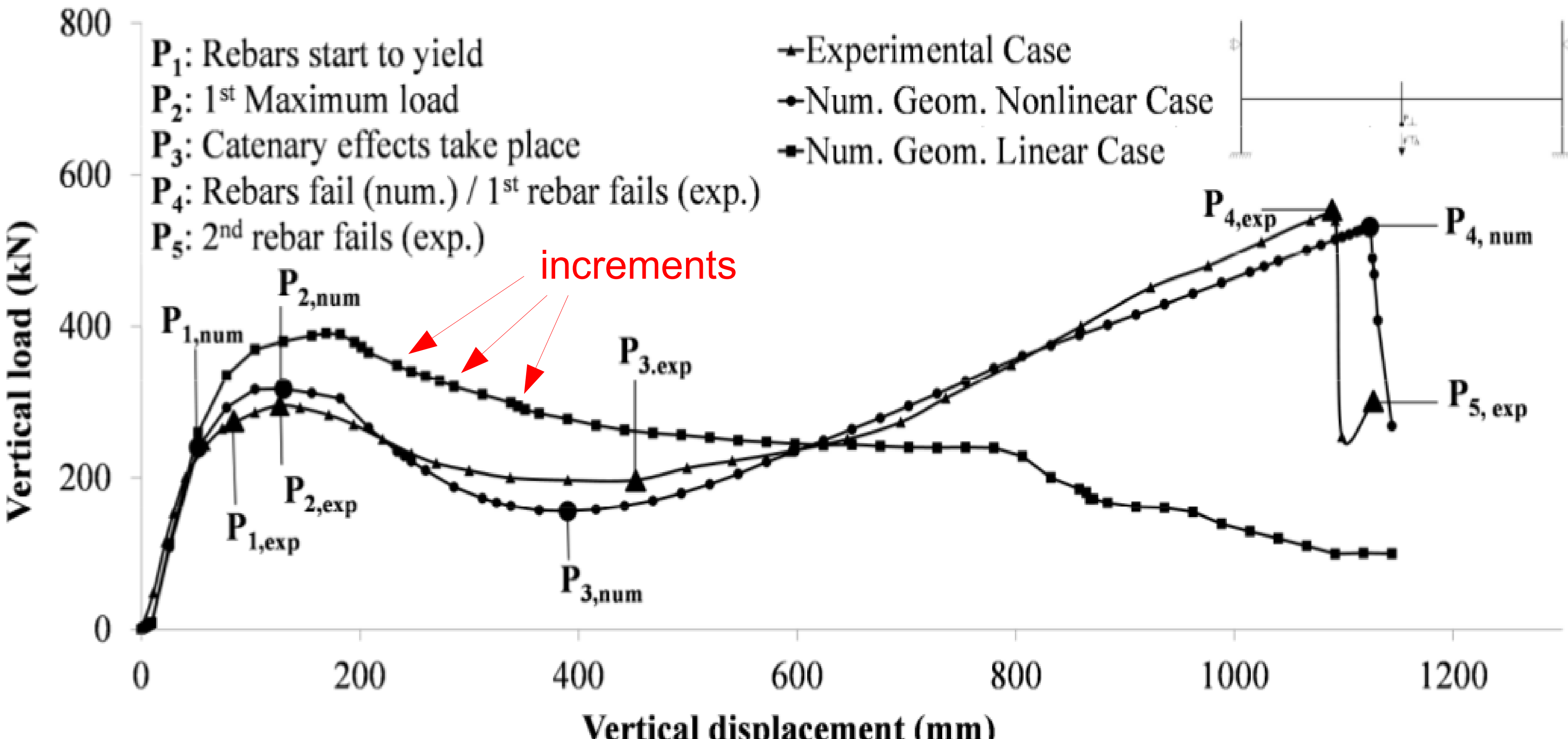
Structure of a NL FE code

Graphical interpretation



Example of nonlinear structural response





Recommended literature

O.C. **Zienkiewicz and** R.L. **Taylor**, The Finite Element Method. **Volume 1**: The Basis. Butterworth-Heinemann, Linacre House, Jordan Hill, Oxford OX2 8DP, 225 Wildwood Avenue, Woburn, MA 01801-2041, England, 2000.

O.C. **Zienkiewicz and** R.L. **Taylor**, The Finite Element Method. **Volume 2**: Solid Mechanics. Butterworth-Heinemann, Linacre House, Jordan Hill, Oxford OX2 8DP, 225 Wildwood Avenue, Woburn, MA 01801-2041, England, 2000.

M.A. **Crisfield**, Non-linear Finite Element Analysis of Solids and Structures **Volume 1**: ESSENTIALS. John Wiley & Sons Ltd. Bafins Lane, Chichester West Sussex PO19 IUD, England, 1991.