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Geometrically nonlinear finite element modelling of linear elastic truss structrures

Péter Z. Berke

1.6. Summary

Inspired and adapted from the 'Nonlinear Modeling of Structures' course of Prof. Thierry J. Massart at the ULB

Outline

Sources of nonlinearities

Geometrical nonlinearities

- Strain and stress measures in FD

Material nonlinearities

Solution procedures

- Newton-Raphson procedure

Advanced subjects

Case studies

Sources of nonlinearities

Translational equilibrium $\sigma_{ij,j} + f_i = 0$

Rotational equilibrium $\sigma_{[ij]} = 0$

Displacement boundary conditions $u_i = \overline{u}_i \text{ sur } S_u$

Surface equilibrium $\overline{T}_i^{(n)} = \sigma_{ij} n_j \text{ sur } S_T$

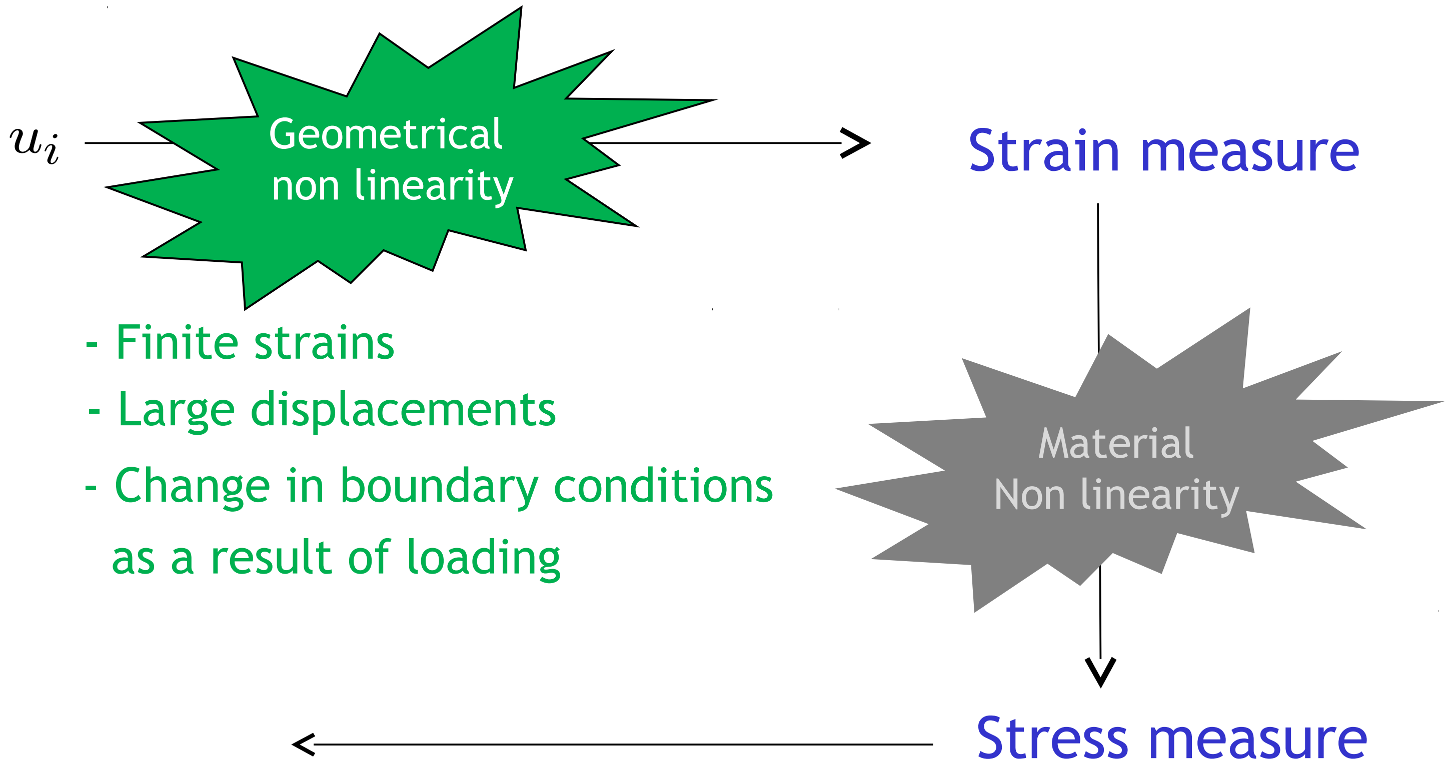
Strain-displacement relationship $\text{defo}_{ij} = f_{NL}\left(u_{i,j}\right)$

Constitutive equations $\sigma_{ij} = g_{NL}\,(\text{defo})$

Geometrical nonlinearities

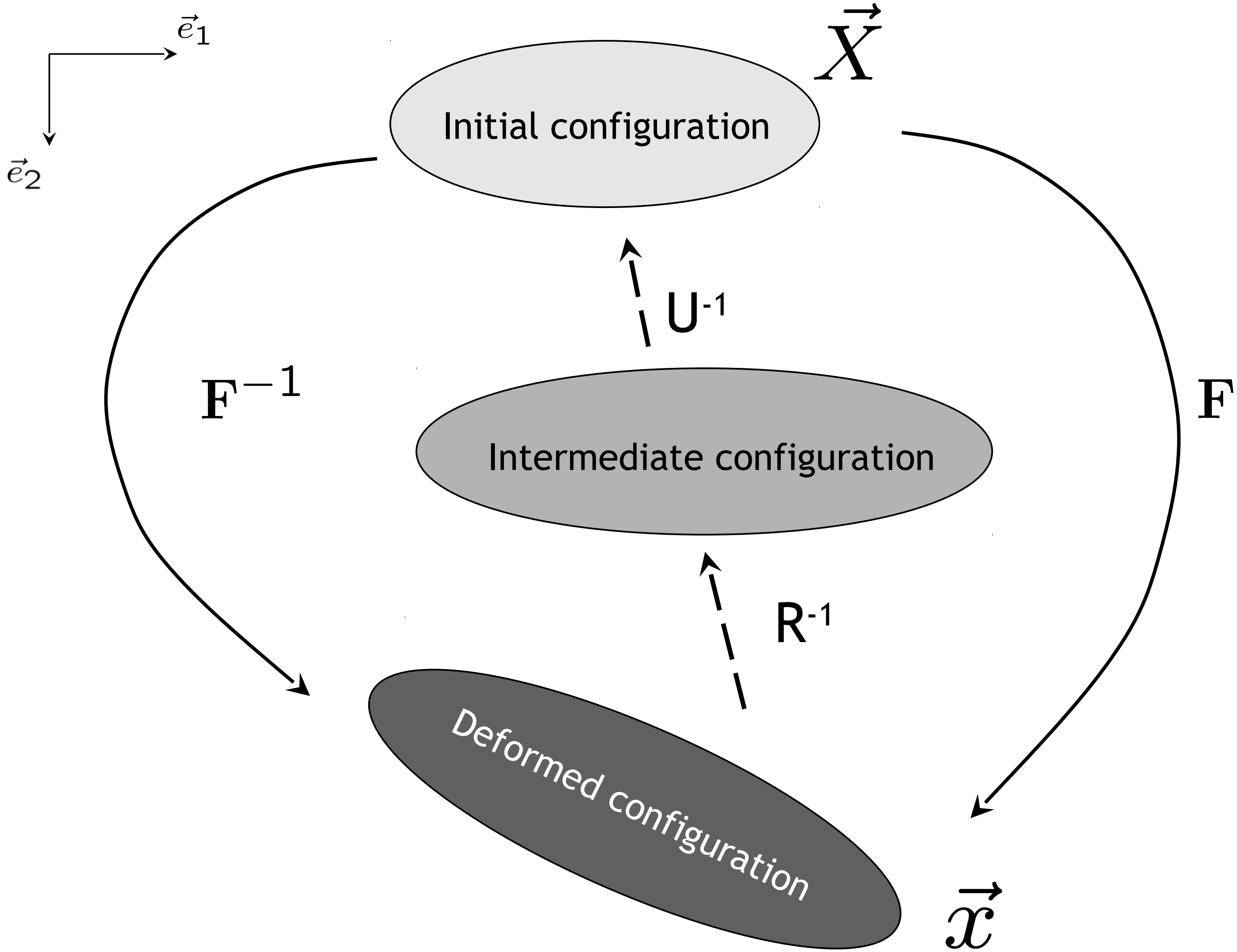
Cause of a non proportionnality between applied forces and
Resulting displacements

Sources



- Finite strains
- Large displacements
- Change in boundary conditions as a result of loading

F = RU



$\vec{e}_1$
$\vec{e}_2$
$\vec{X}$
Initial configuration
U^{-1}
$\mathbf{F}^{-1}$
Intermediate configuration
$\mathbf{F}$
R^{-1}
Deformed configuration
$\vec{x}$

Geo NL - Strain measures

Different tensors can be defined

- Biot strain $\mathbf{E}^B = \mathbf{U} - \mathbf{I}$
- Logarithmic strain $\mathbf{E}^N = \ln \mathbf{U}$
- Euler strain tensor $\mathbf{E}^E = \frac{1}{2}\left(\mathbf{I} - \mathbf{U}^{-2}\right)$
- Green strain tensor $\mathbf{E}^G = \frac{1}{2}\left(\mathbf{U}^2 - \mathbf{I}\right) = \frac{1}{2}\left(\mathbf{F}^T \mathbf{F} - \mathbf{I}\right)$

The infinitesimal strain tensor is NOT objective!

$$\cancel{\varepsilon_{jk} = \frac{1}{2}\left(\frac{\partial u_k}{\partial X_j} + \frac{\partial u_j}{\partial X_k}\right)}$$

Geo NL - Principle of virtual work

$$\delta W_{int} = \delta W_{ext}$$

with

$$\delta W_{int} = \int_{vol} (\text{stress}) : \delta(\text{conjugate strain}) d(vol)$$

$$\delta W_{ext} = \int_V \vec{f}.\delta\vec{u}dV + \int_S \vec{p}.\delta\vec{u}dS$$

Conjugate quantities

By definition, two stress and strain quantities are conjugate if their internal product integrated on the proper configuration gives the correct internal work

Solution procedures

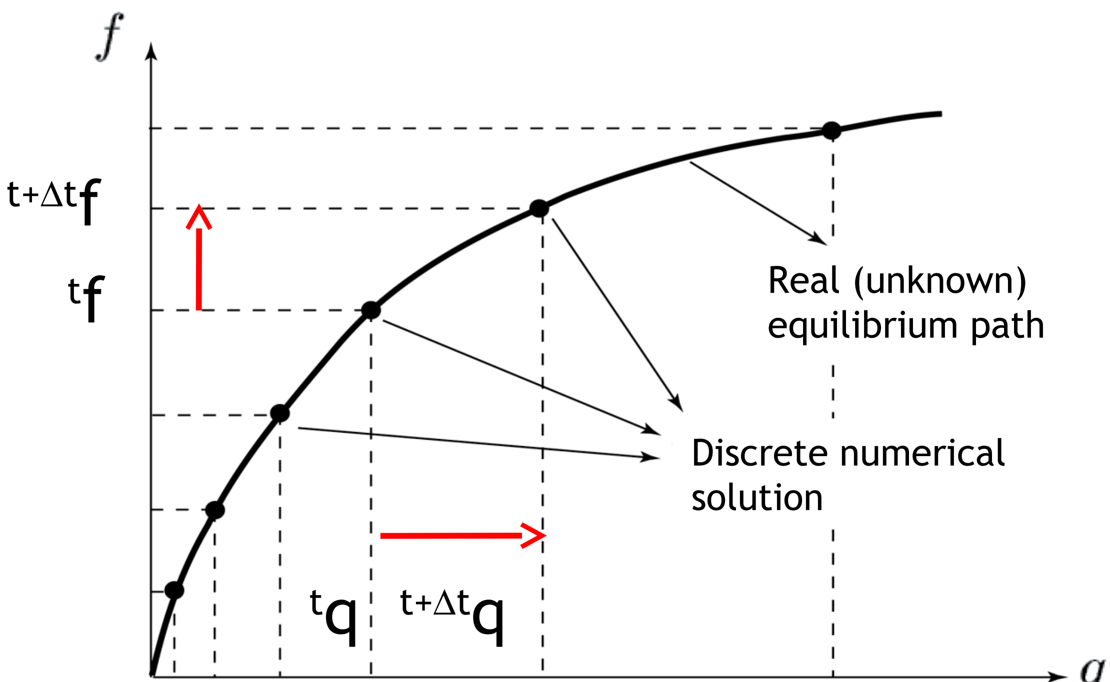
Why working with increments ?

The behaviour of materials may depend on the path followed

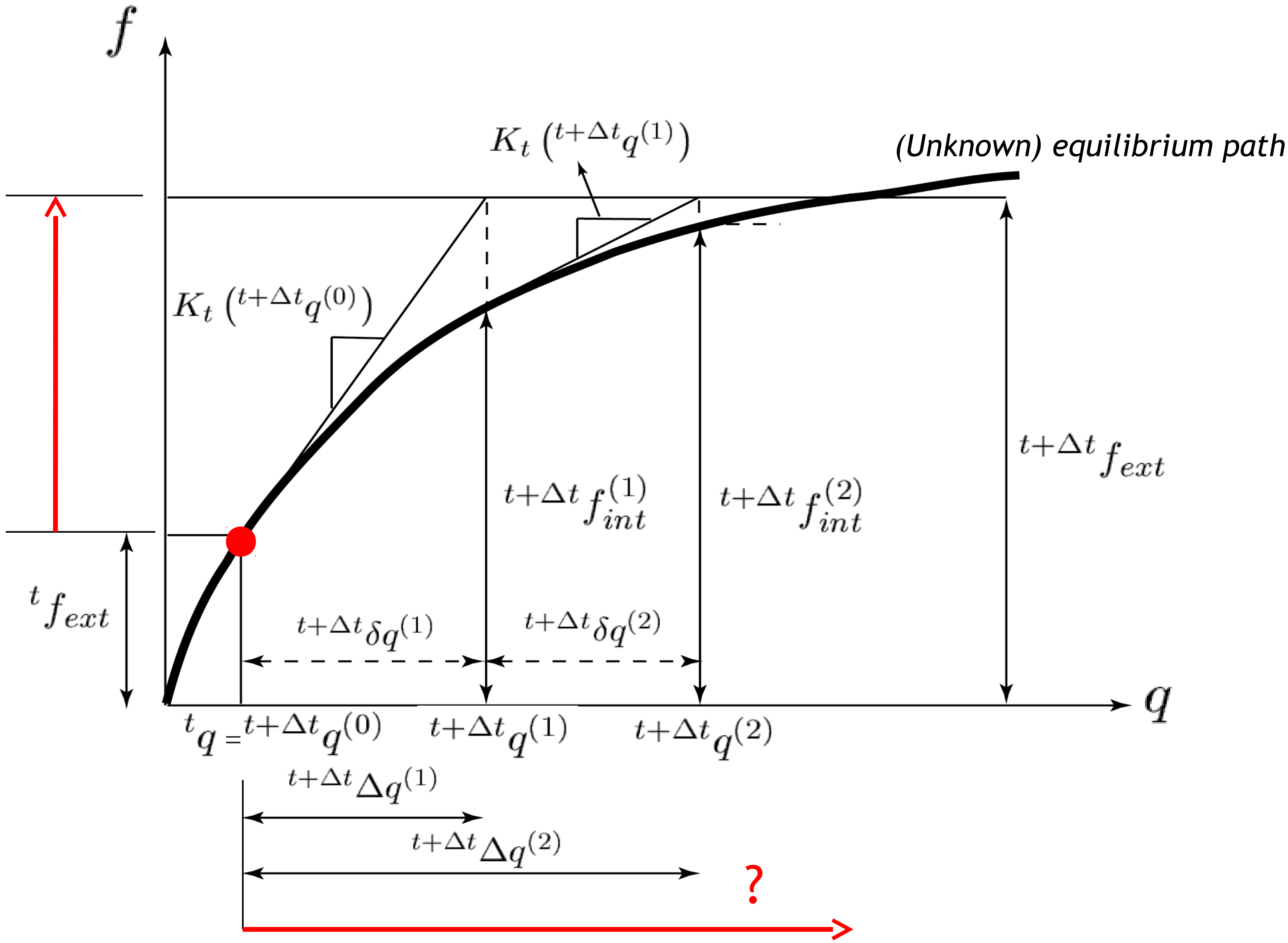
The radius of convergence of iterative procedures (NR) is limited

$\Rightarrow$ Apply loads with discrete, successive steps

$\Rightarrow$ The structural response is evaluated at discrete points

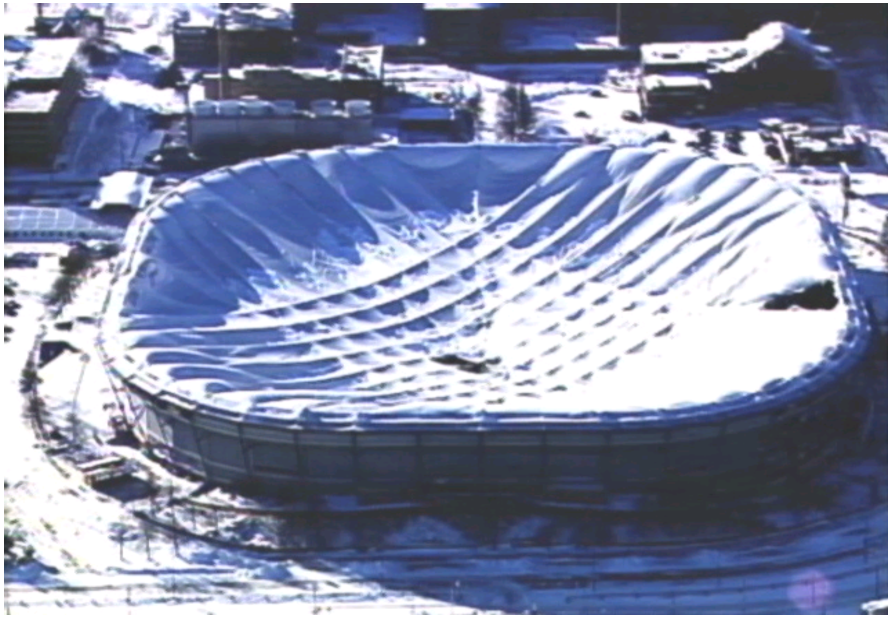


Incremental-iterative procedure



Metrodome, Minneapolis, USA
December 2010

[http://www.themegaworldnews.com/wp-content/uploads/2010/12/dome_pic2.jpg]



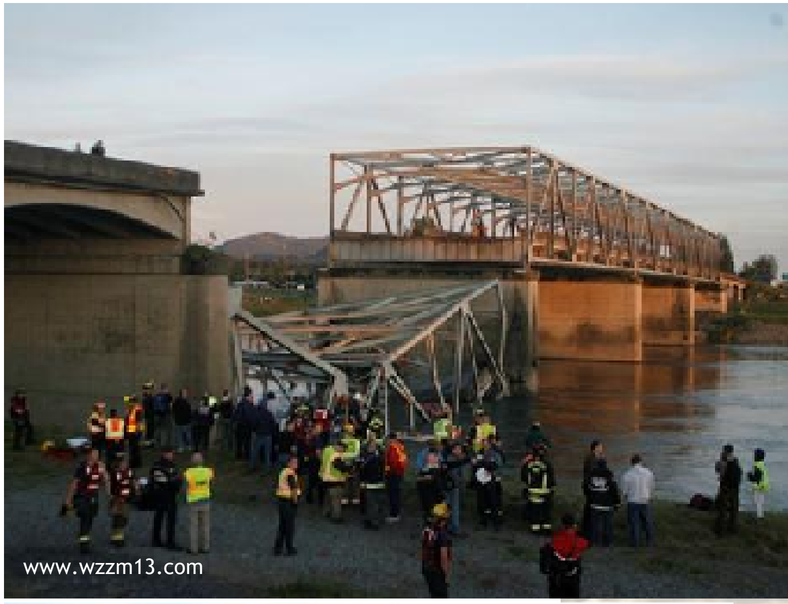
[http://cdn.bleacherreport.net/images_root/images/photos/001/087/887/metrodome-269x198_crop_340x234.jpg?1292257653]

Case study #2

Seattle, 24th May 2013

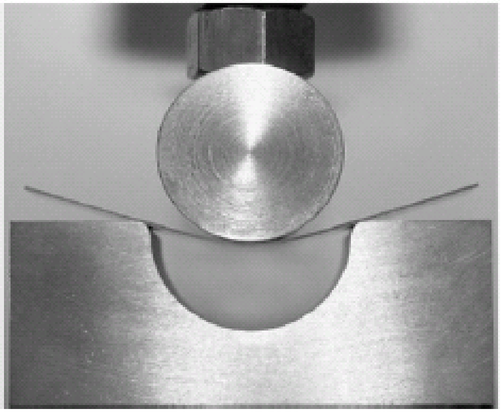
Washington bridge

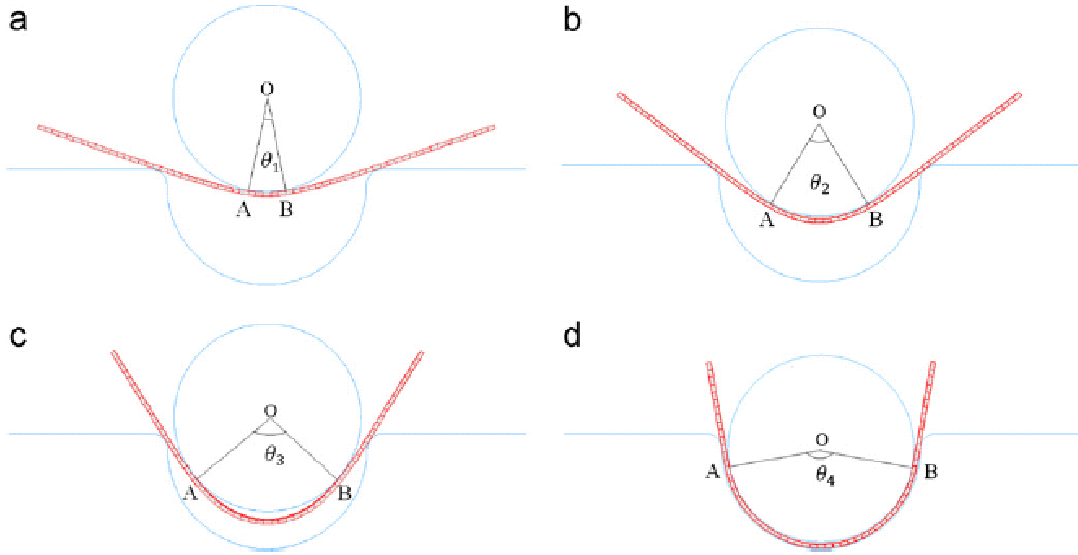
http://www.foxnews.com/us/2013/05/24/highway-bridge-collapses-in-washington-state-people-in-water/

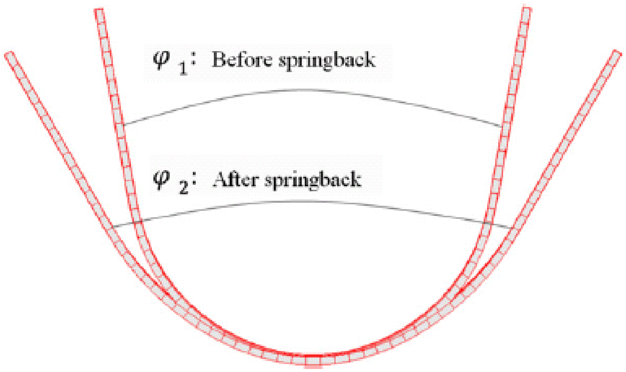


www.nydailynews.com

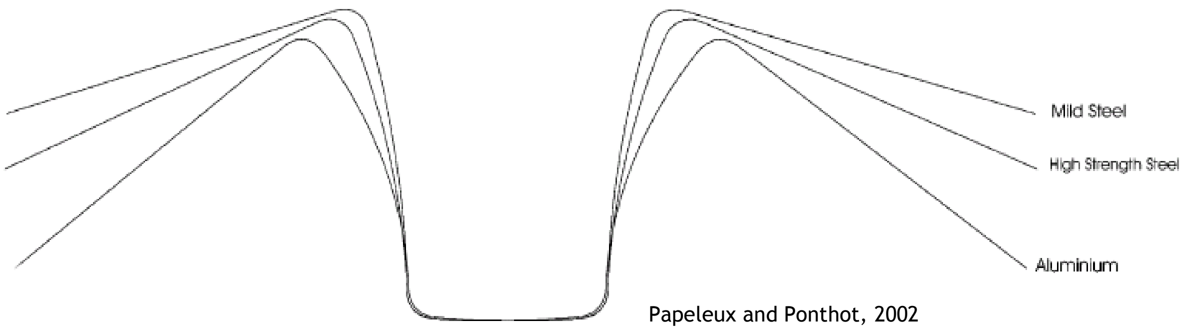
Sheet metal forming - Elastic springback?







Xu et al., 2012



Papeleux and Ponthot, 2002

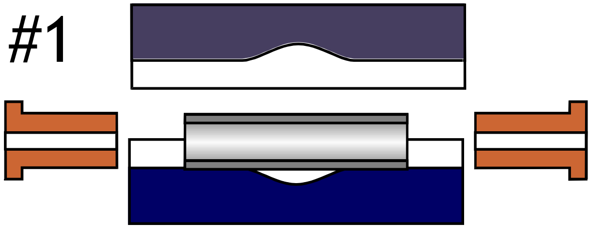
Case study #4

[http://baetlanguedoc.blog50.com/archive/2009/01/30/nouvelle-generation-de-pneus.html]

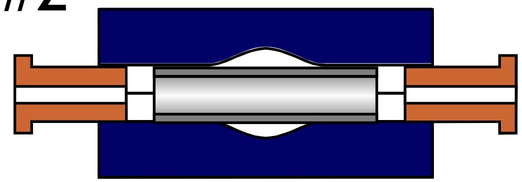
Case study #5

Hydro-forming

#1



#2



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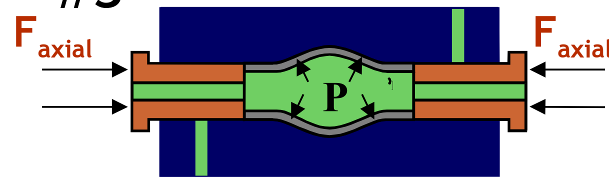

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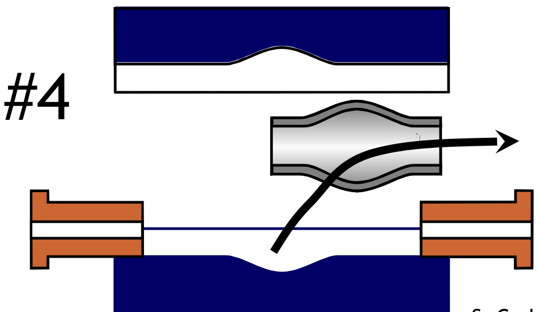
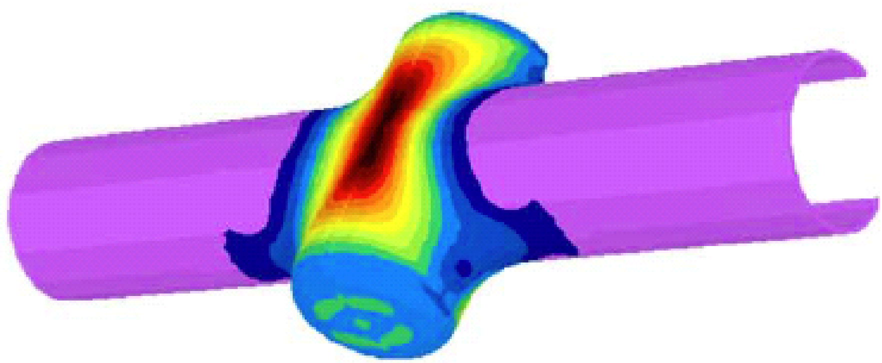


S. Godet



Chenot and Massoni, 2006

Case study #6

Complexities, necessary physical ingredients?

http://www.eef.org.uk/NR/rdonlyres/CABEE2C5-271E-419B-876A-6E8ED573A15F/12685/hot4.jpg

Case study #7

[http://cache2.allpostersimages.com/p/LRG/26/2634/EPCMD00Z/poster/cork-flying-out-of-champagne-bottle.jpg]

Case study #7

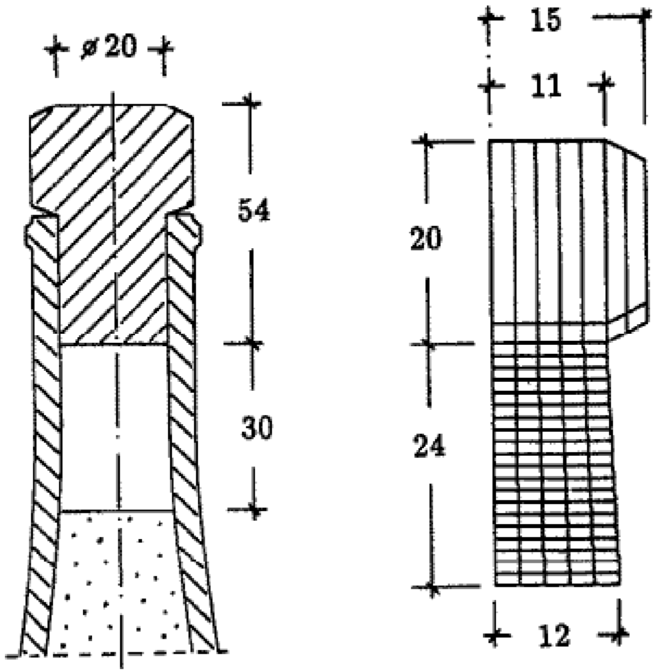


Fig. 14. Finite element model of the cork.

[Wriggers, 1989]

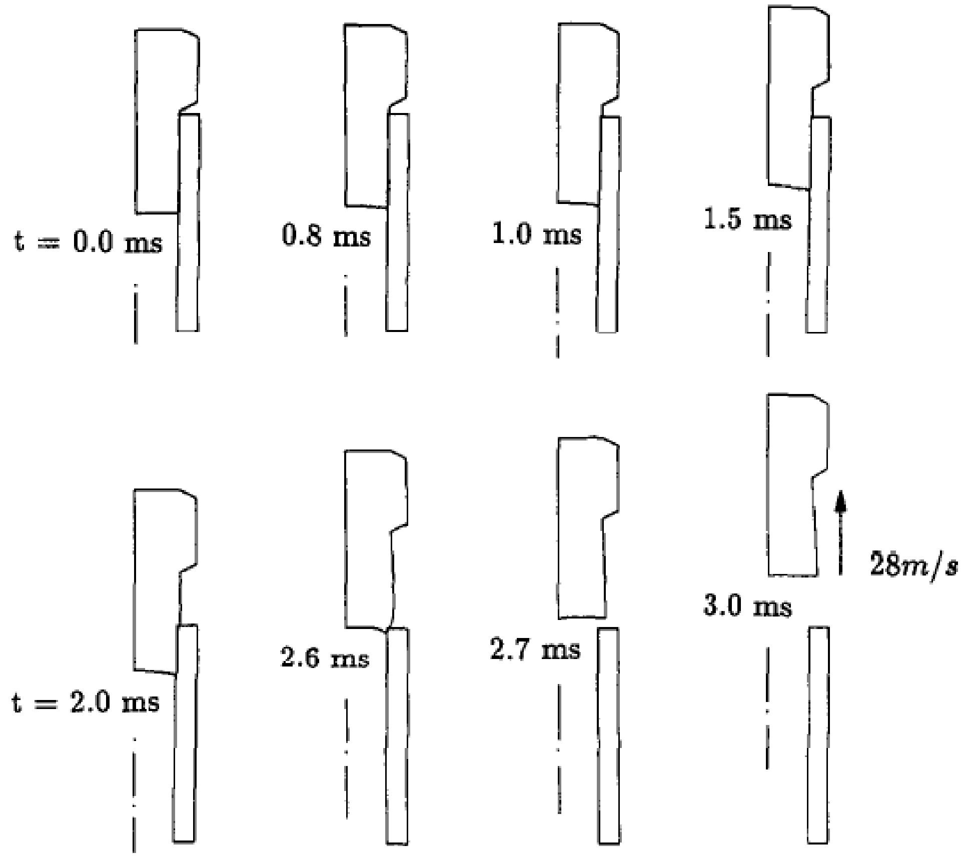


Fig. 15. The configurations of the cork at several time instants (0.0, 0.8, 1.0, 1.5, 2.0, 2.6, 2.7 and 3.0 msec).