Nonlinear Multi-Scale Finite Element Modeling of the Progressive Collapse of Reinforced Concrete Structures

Lecture 6: Multiscale methodology

Péter Z. Berke

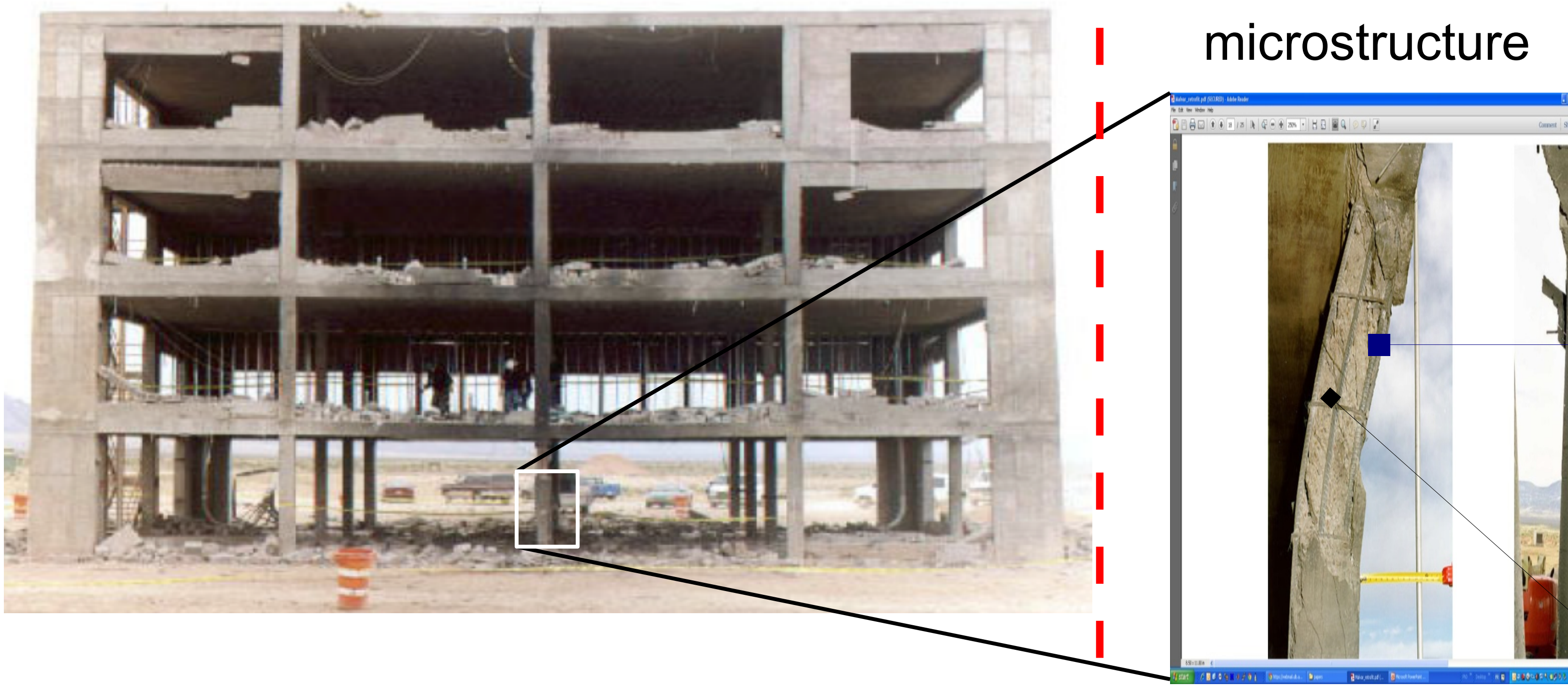
Visiting Professor
Departamento de Engenharia Metalúrgica e de Materiais
Universidade Federal do Ceará, Bloco 729
Scientific Collaborator
Building, Architecture and Town Planning Dept. (BATir)
Université Libre de Bruxelles (ULB), Brussels, Belgium

Inspired and adapted from the 'Nonlinear Modeling of Structures' course of Prof. Thierry J. Massart at the ULB

Scales of representation

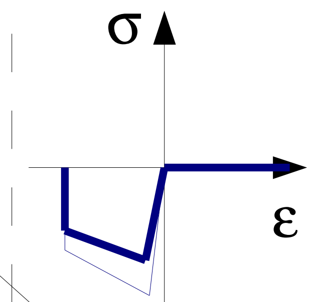
Structure

Material microstructure

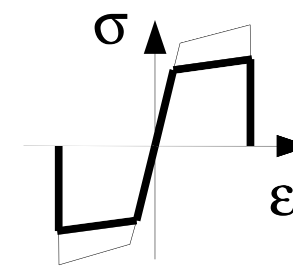


Material behavior

concrete



steel



Homogenization techniques

Obtain the mechanical properties of a material at a higher scale from a microstructural element, i.e. the characteristics of

- the material's structure (texture, different phases)
- the behavior of its phases (constitutive laws)

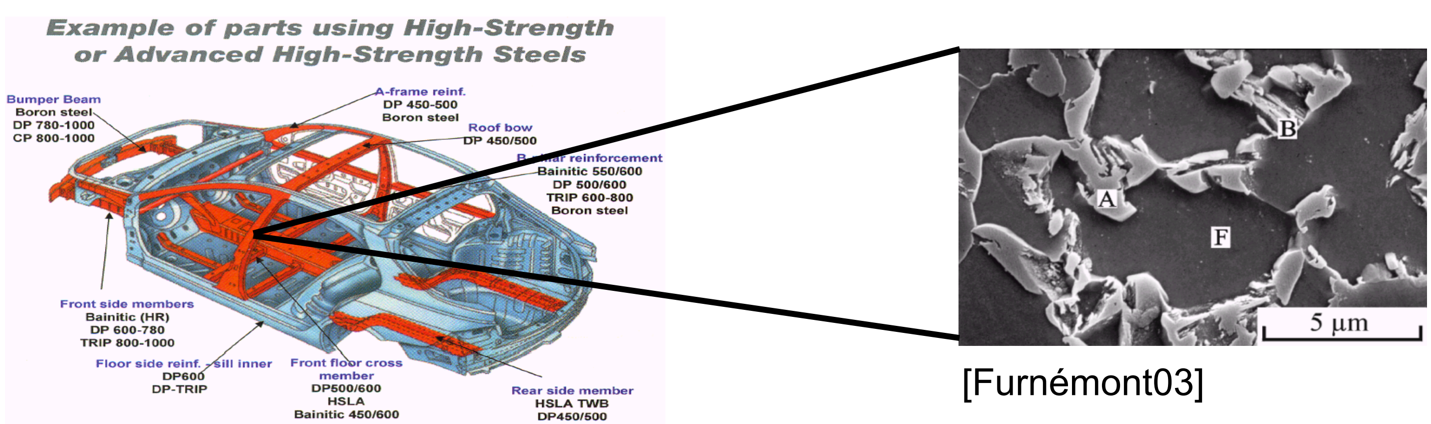
Principles of homogenization

The material is considered as a structure

Scale transition operators need to be defined

Used for :

- Material identification
  - Identification of the behavior of separate phases
  - Global behavior of the composite obtained by homogenization
- Microstructure optimization
  - Optimize material properties of the composite
- Replace complex macroscopic constitutive laws
  - Principle of multiscale methods



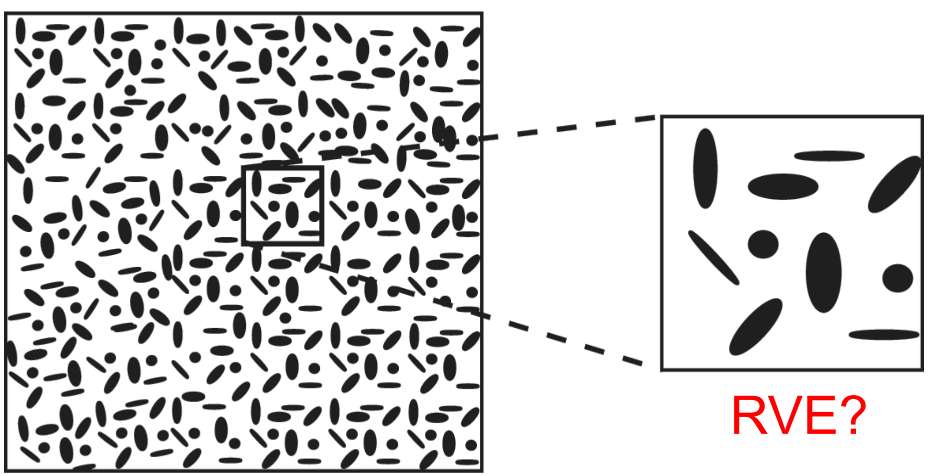
[Furnémont03]

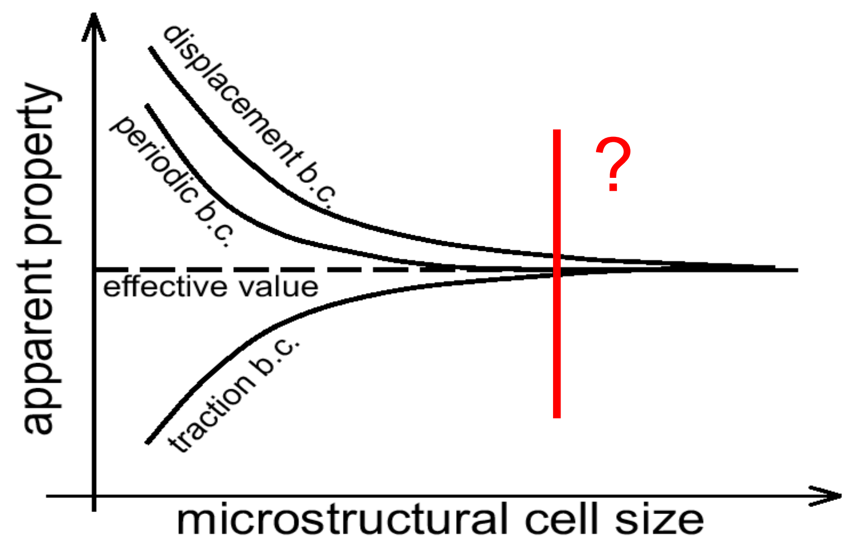
Representative Volume Element (RVE)

Matrerial considered as a structure

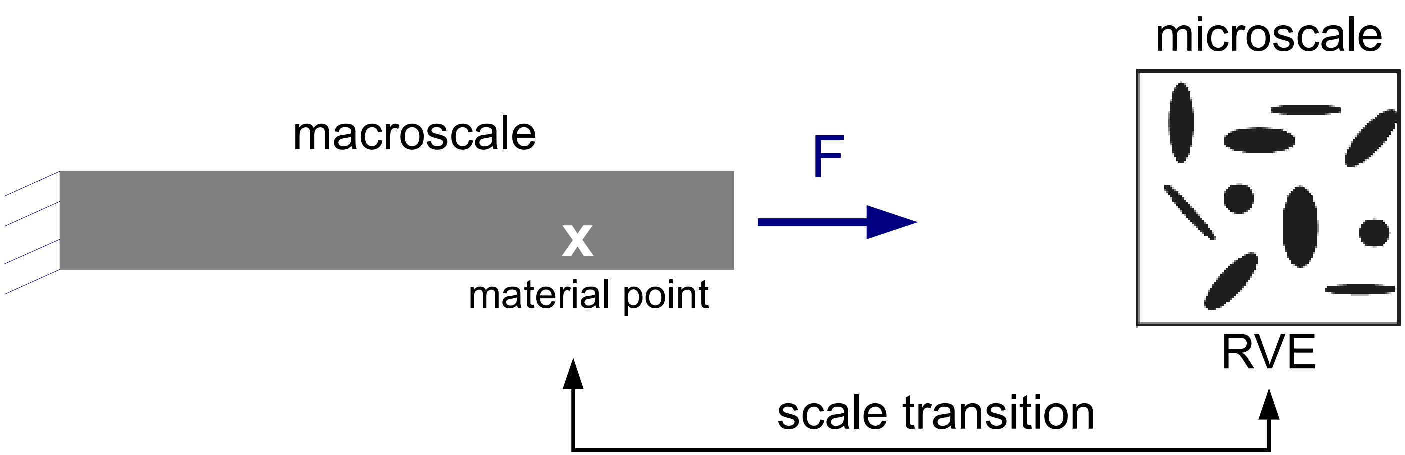
Extraction of a volume representative of the microstructure

- Sufficiently large to be statistically representative
- Sufficienly small for reasonable computational time
- Depends on the material behavior to consider





Equivalence relationships between micro- and macroscales



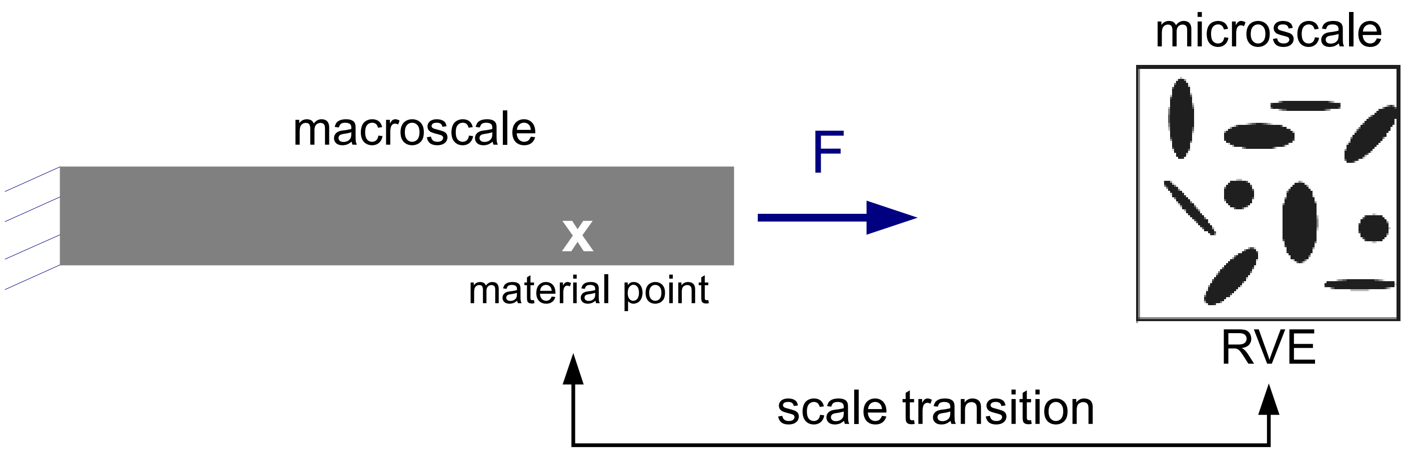
$$\mathbf{E} = \frac{1}{V}\int_V \boldsymbol{\varepsilon}\, dV \qquad \boldsymbol{\Sigma} = \frac{1}{V}\int_V \boldsymbol{\sigma}\, dV$$

Macro strain — Micro strain — Macro stress — Micro stress

$$\boldsymbol{\Sigma} : \mathbf{E} = \frac{1}{V}\int_V \boldsymbol{\sigma} : \boldsymbol{\varepsilon}\, dV$$

Energy equivalence (Hill-Mandel)

Equivalence relationships between micro- and macroscales



Strain equivalence

$$\langle \varepsilon \rangle = \mathbf{E} + \frac{1}{V} \oint_S (\vec{w}\vec{n})^{sym} \, dS$$

$$\rightarrow \quad \oint_S (\vec{w}\vec{n})^{sym} \, dS = 0$$

$$\vec{u}\,(\vec{x}) = \mathbf{E}.\vec{x} + \vec{w}$$

Macro strain ($\mathbf{E}$)

Microfluctuation (heterogeneities) ($\vec{w}$)

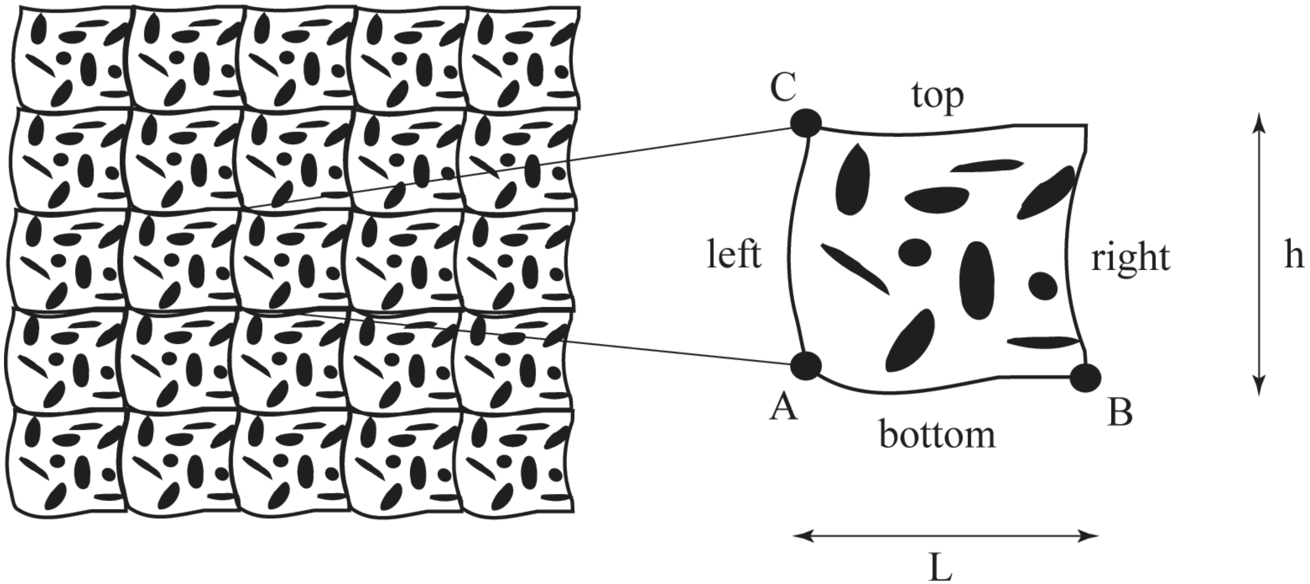
Microfluctuation supposed to be periodic

Assumption of local periodicity

- macroscale quantities imposed at 3 corners of the RVE

$$\mathbf{E} = \frac{1}{\mathrm{V_{cell}}} \int_{\mathrm{V_{cell}}} \boldsymbol{\varepsilon} \ \mathrm{dV} = f\left(\vec{u}_A, \vec{u}_B, \vec{u}_C, h, L\right)$$

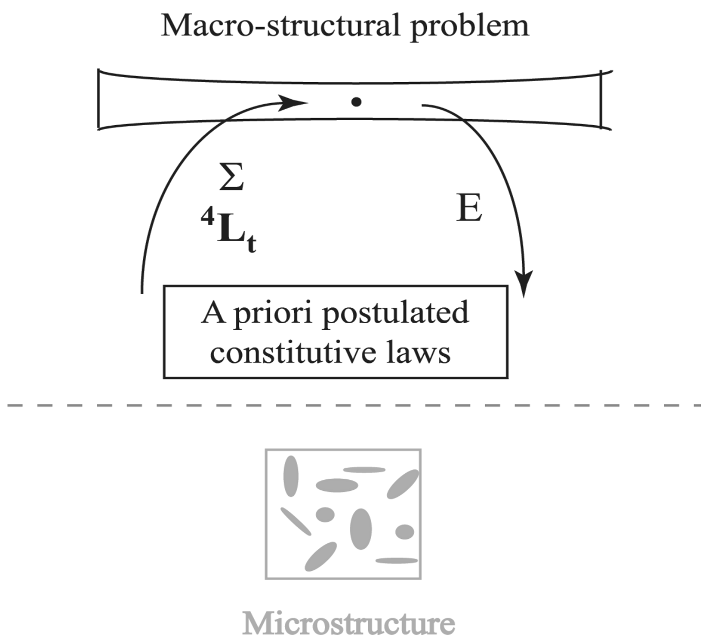
$$\boldsymbol{\Sigma} = \frac{1}{V} \int_V \boldsymbol{\sigma} dV = \sum_{i=A,B,C} \vec{f}^{(i)} \vec{x}^{(i)}$$



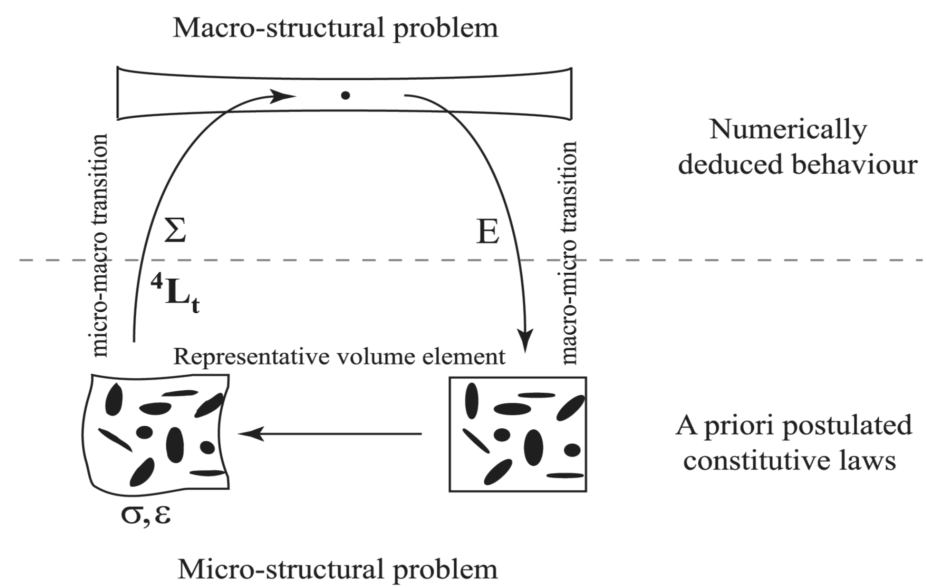
$$\vec{u}_{\mathrm{right}} = \vec{u}_{\mathrm{left}} + \vec{u}_{\mathrm{B}} - \vec{u}_{\mathrm{A}} \qquad \vec{u}_{\mathrm{top}} = \vec{u}_{\mathrm{bottom}} + \vec{u}_{\mathrm{C}} - \vec{u}_{\mathrm{A}}$$

Principle of multiscale modeling

Classical approach



Multiscale approach



Phenomenological material law:

- Convoluted effect of the microstructure
- Validity for all type of loading history
- Experimental identification

Computationally derived behavior:

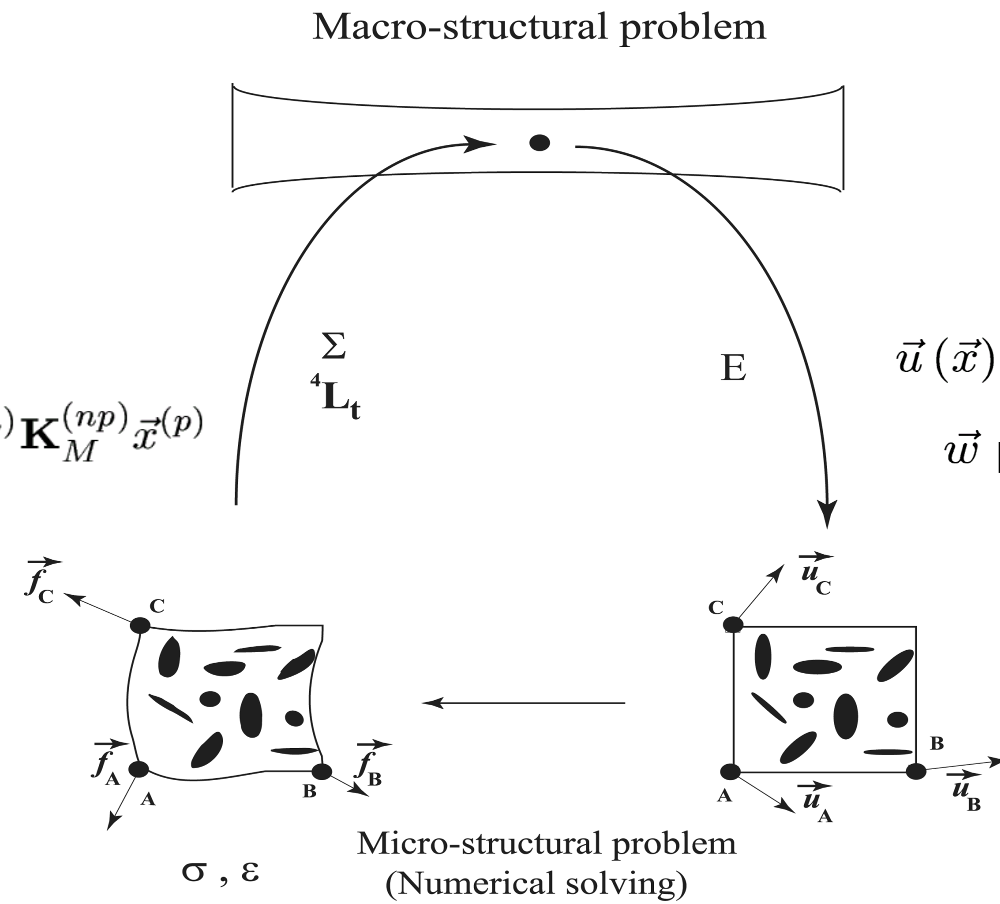
- Identification of the scale for phen. laws
- Define RVE for the microstructure

-Formulate scale transition

First order multiscale scheme

$$\boldsymbol{\Sigma} = \frac{1}{V_{\mathrm{RVE}}} \sum_{i=A}^{C} \vec{x}^{(i)} \vec{f}^{(i)}$$

$${}^{4}\mathbf{L}_t = \frac{1}{V_{\mathrm{RVE}}} \sum_{n=A}^{C} \sum_{p=A}^{C} \vec{x}^{(n)} \mathbf{K}_M^{(np)} \vec{x}^{(p)}$$



$$\vec{u}\,(\vec{x}) = \mathbf{E}.\vec{x} + \vec{w}$$

$\vec{w}$ periodic

Condensed tangent stiffness

$$\delta f^{(i)} = \mathbf{K}_M^{(ij)} \delta u^{(j)} \qquad i, j = A, B, C$$

Microscale tangent stiffness

$$\delta f^{(i)} = \mathbf{K}_m^{(ij)} \delta u^{(j)} \qquad i, j = 1, ..., nb_{dofs}$$

Summary

+

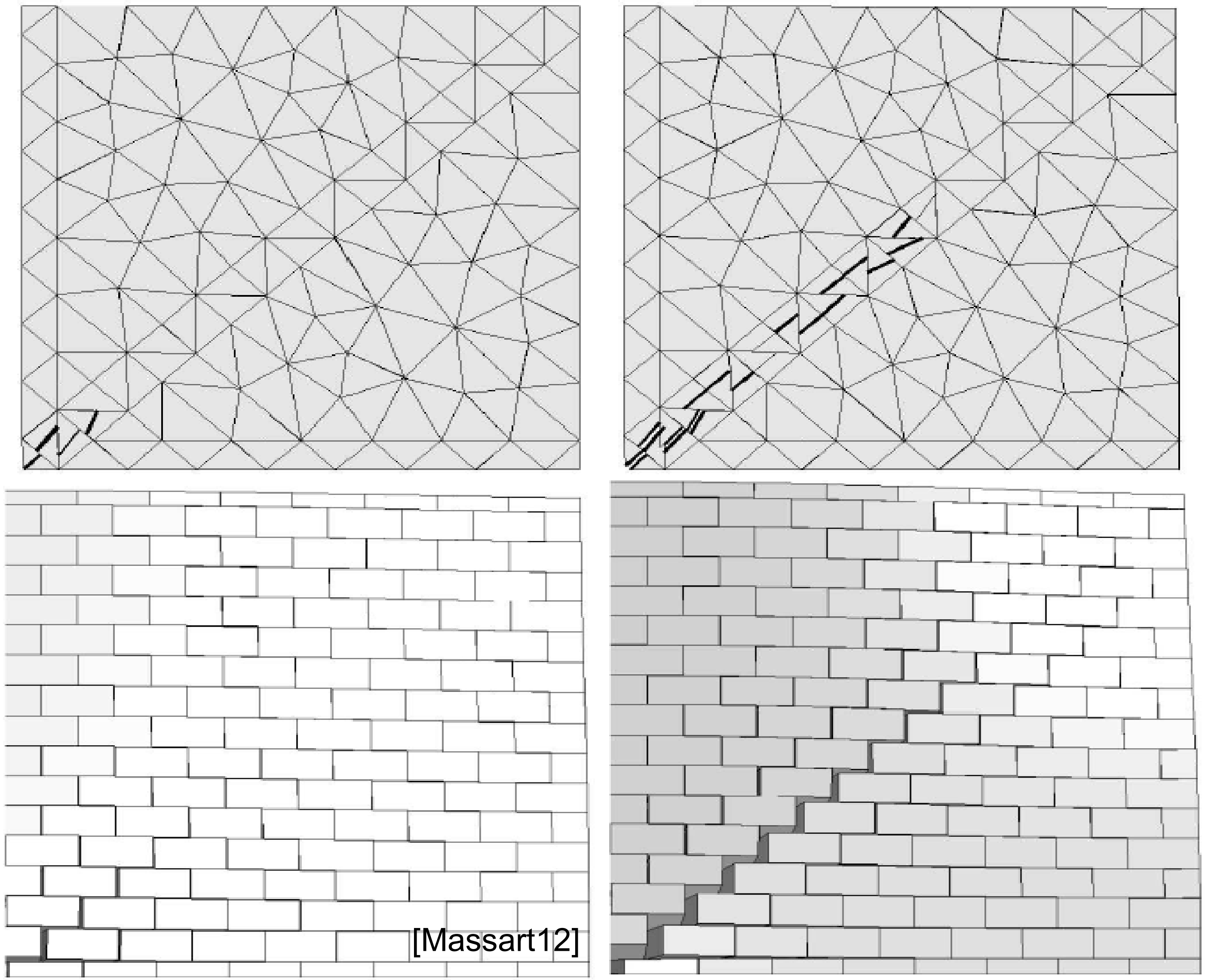
- Avoids the use of complex macroscale constitutive law
- Identification on homogeneous phases on the microscale
- Interaction between phases is taken naturally into account
- Finite strains and complex loading path
- Independent from the micromechanical model
- Easy integration into commercial FE codes

-

- Very high computational cost, but....
  possibility of a parallel solver
  selective scale transition to the microscale?
  simple micromechanical models, complex macroscale response
- Principle of local action MUST be valid

$$\text{periodicity} \implies \text{size}_{\text{RVE}} \ll \text{size}_{\text{macro-point}}$$

Example – Masonry cracking



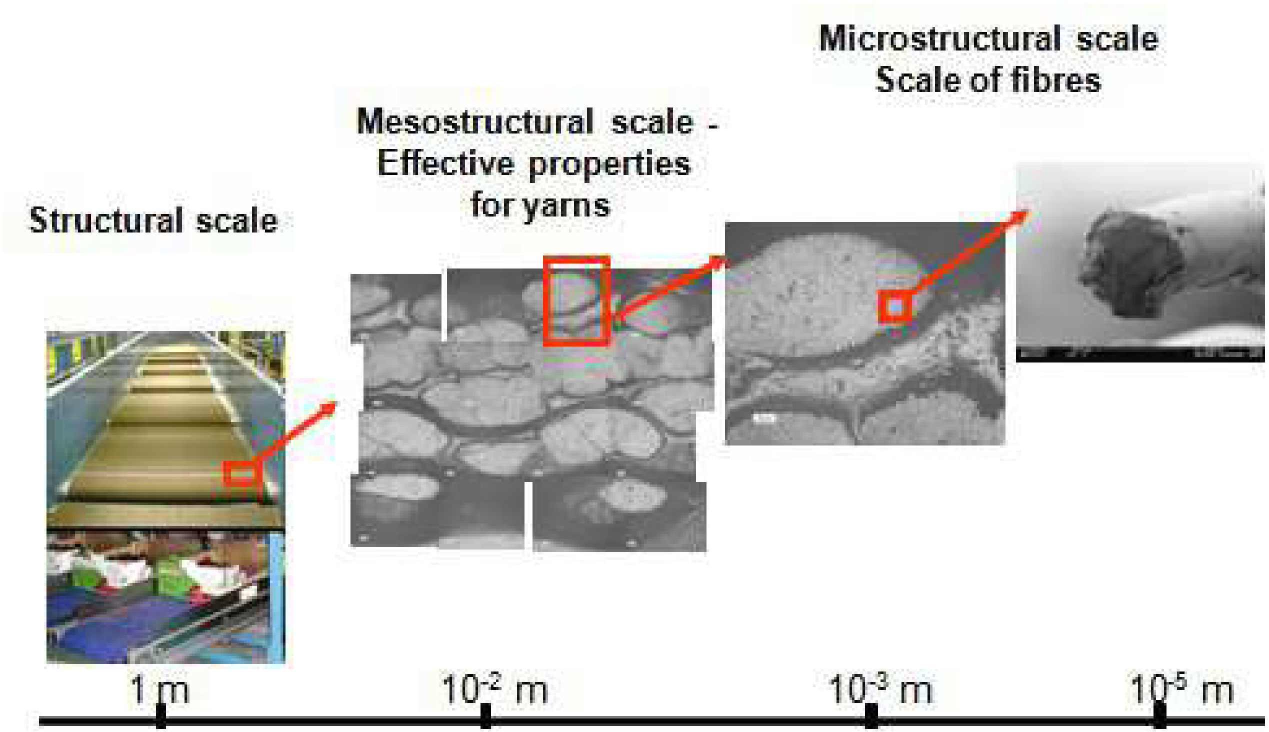
w = 0.5 mm

w = 0.85 mm

Multiscale approach

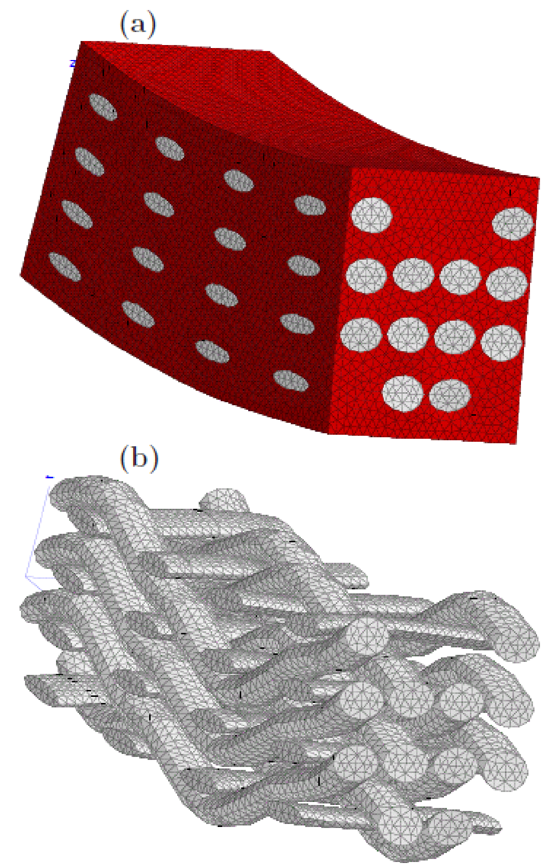
UNIVERSIDADE FEDERAL DO CEARÁ

Example – 3D woven composite



Computational homogenization

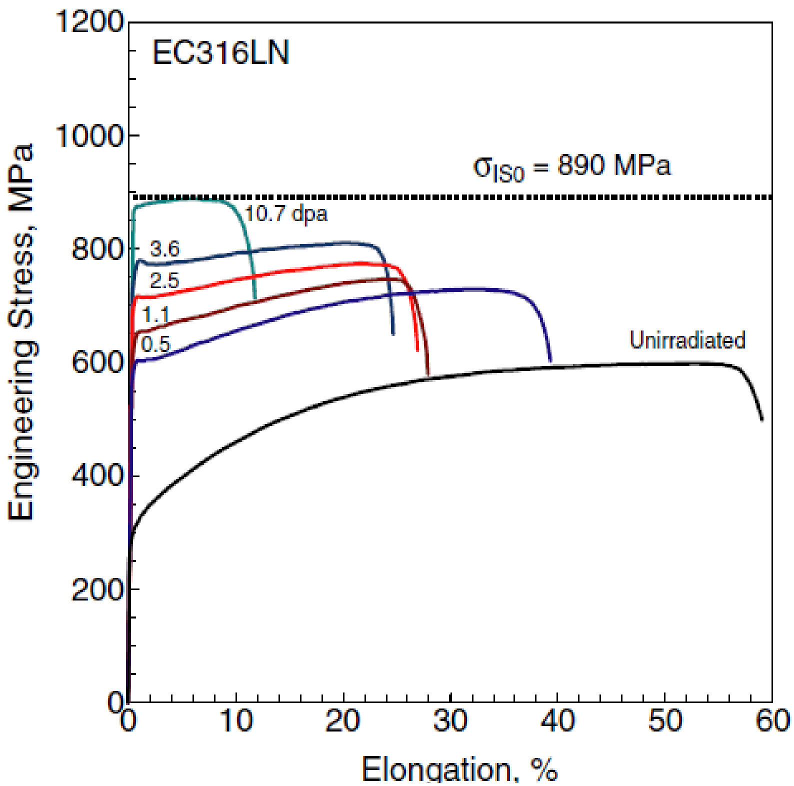
conveyor belt RVE

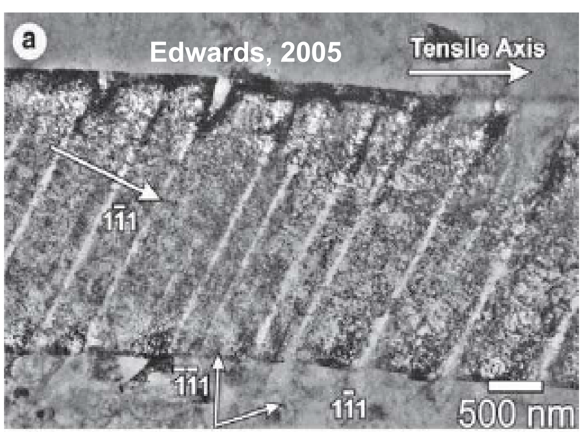


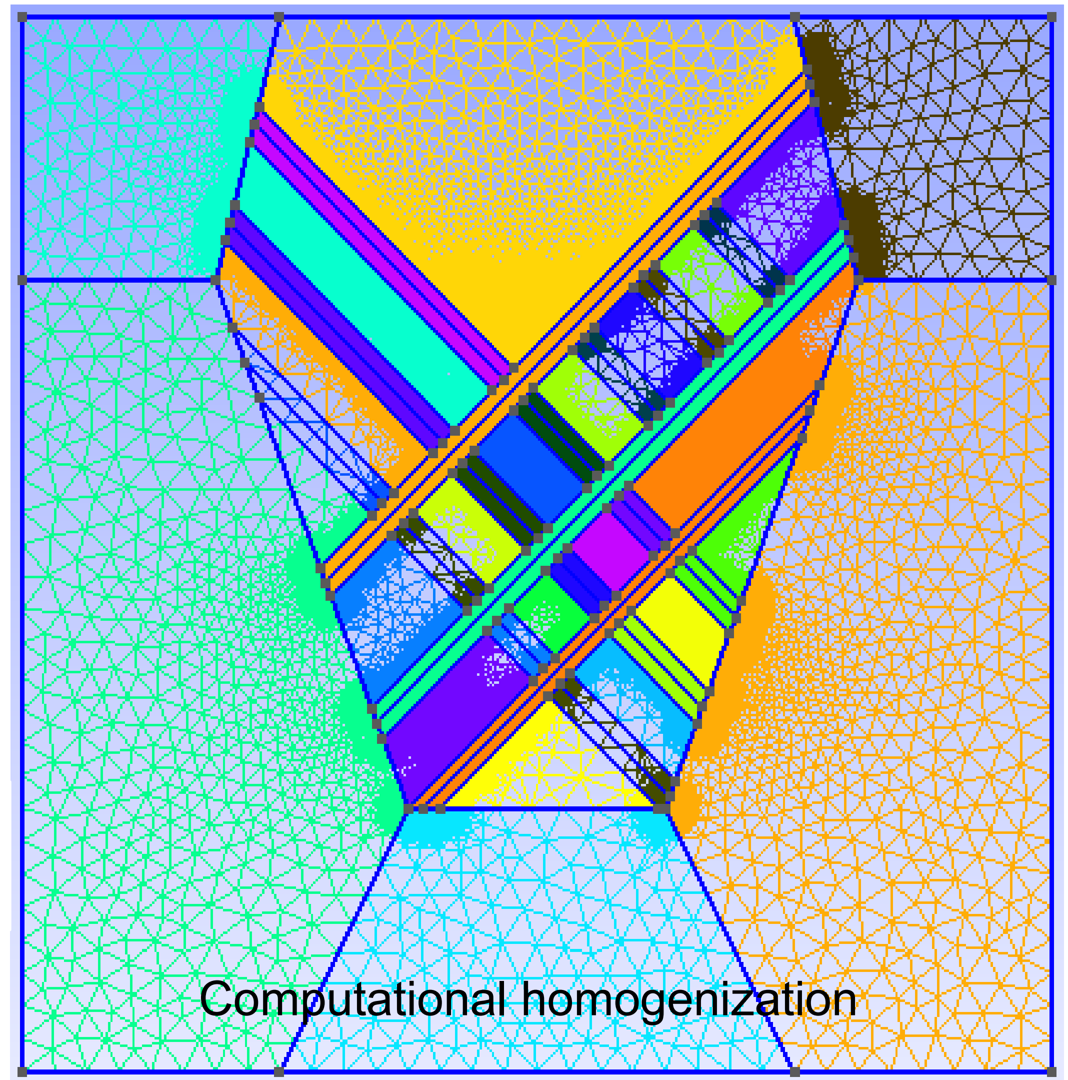
[Piezel12]

Universidade Federal do Ceará

Example – Clear band formation







Steel RVE