Advanced Analysis of Steel Frames

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Outline

- Introduction
- CS-ASA
- Second-Order Effects
- Semi-Rigid Connections
- Inelastic Effects
- Combined Effects
- Nonlinear Solvers
- Nonlinear Dynamic Problem
- Final Remarks

Introduction

Fundamentals: Frame stability and strength
Nonlinear behavior of steel frames under static and dynamic loads

Nonlinear effects: Second-order
Steel's inelasticity
Semi-rigid connections

Nonlinear FE program: CS-ASA (Silva, 2009)

Methods of Analysis

First-order elastic analysis

- Material: linear-elastic
- Equilibrium: undeformed configuration
- Forces and deformations: proportional
- Frame stability: no measure
- Frame strength: no measure (the most simple method !!)

Second-order elastic analysis

- Material: linear-elastic
- Equilibrium: deformed configuration
- Forces and deformations: no proportional
- Frame stability: measure
- Frame strength: no measure

Methods of Analysis

First-order inelastic analysis

- Material: consider plasticity
- Equilibrium: undeformed conf
- Forces and deformations: no
- Frame stability: no measure
- Frame strength: measure (the plastic limit load !!)

Other factors...

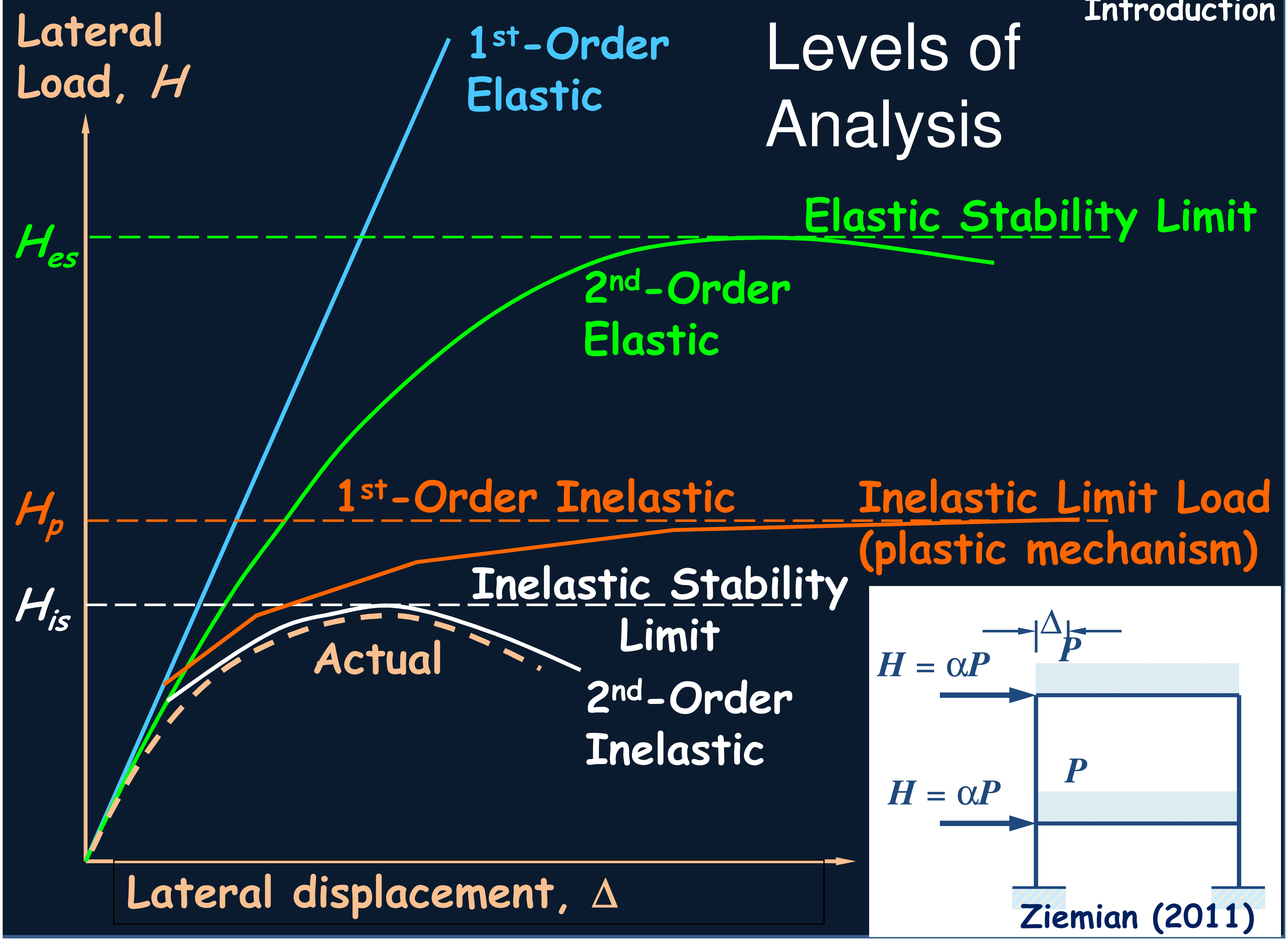
- Connections
- Fabrications and erection tolerances
- Residual stress

+

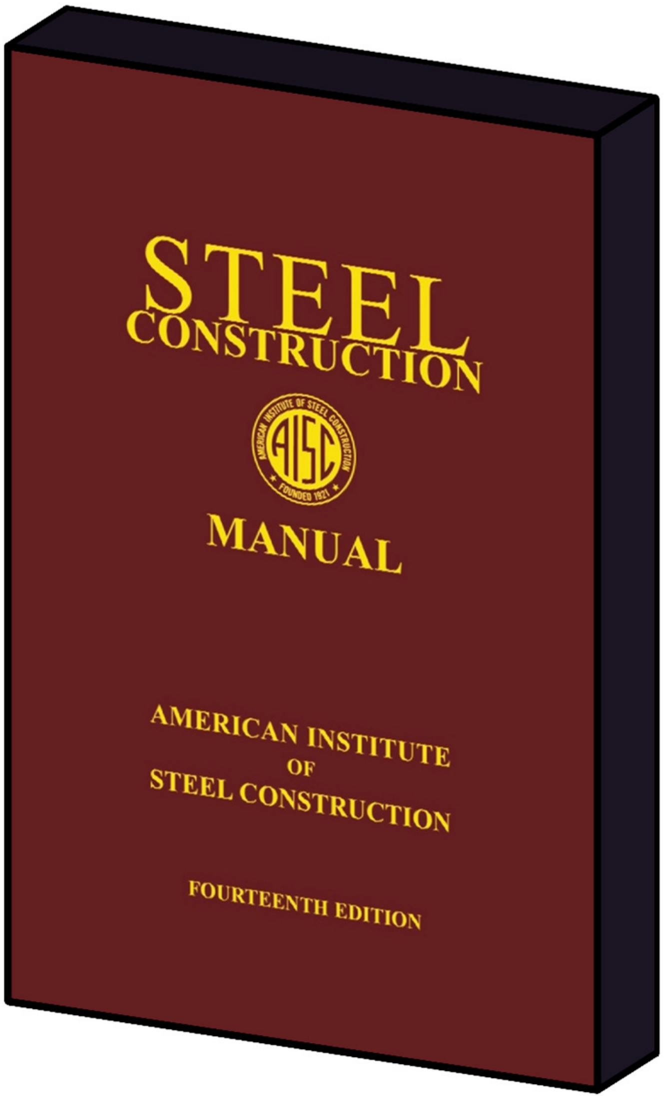
Second-order inelastic analysis

- Material: consider plasticity
- Equilibrium: deformed configuration
- Forces and deformations: no proportional
- Frame stability: measure
- Frame strength: measure

Levels of Analysis



AISC 2010 Specification



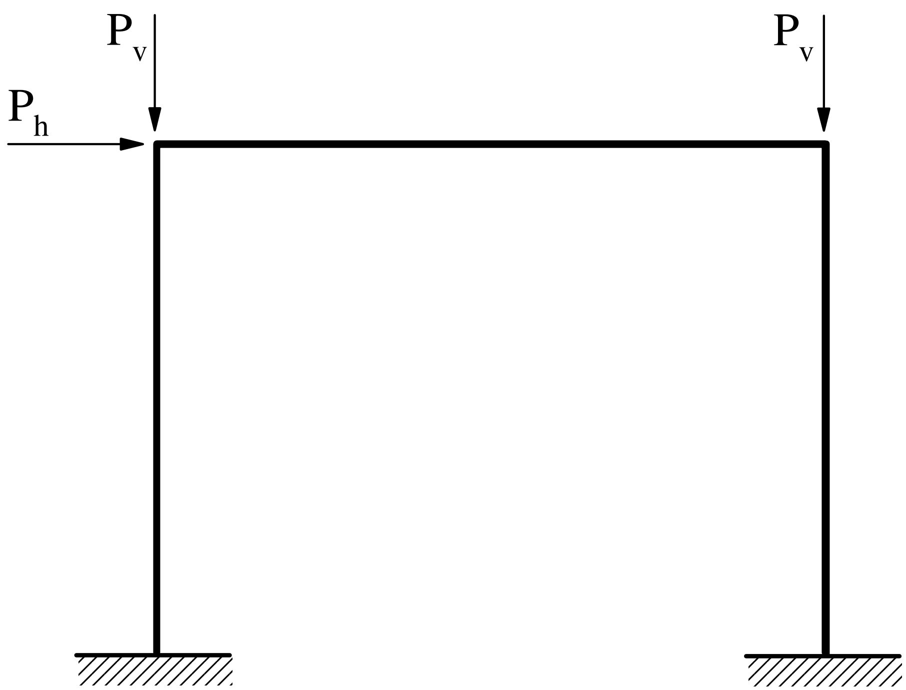
Appendix 1:
Design by Inelastic Analysis

Goals of Appendix 1:

- ✓ Address application of a <u>wide range</u> of current and emerging methods of <u>inelastic analysis</u>
- ✓ Mirror the elastic stability provisions of Ch. C; <u>Direct Analysis Method</u>
- ✓ **Eye toward the future and moving to Performance Based Design**
- ✓ Transparent and reasonably <u>self-contained</u>

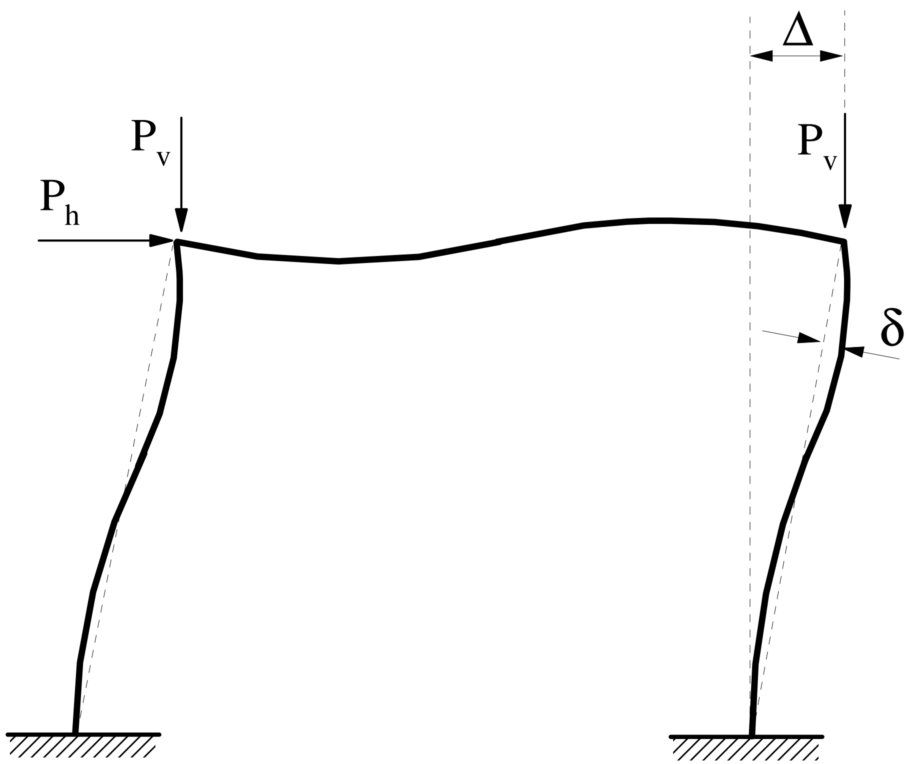
Ziemian (Cilamce, 2011)

Simple Portal Frame



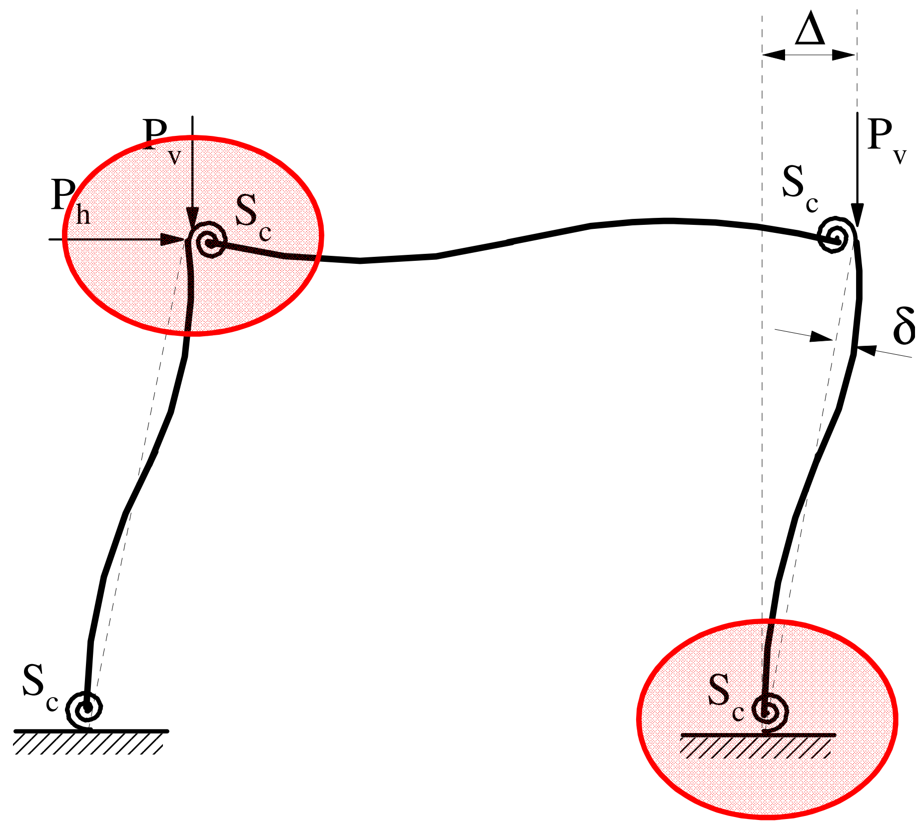
Second-Order Effects

$$\mathbf{F}_{int}\left(\mathbf{U},\mathbf{P}\right)=\mathbf{F}_{ext}$$



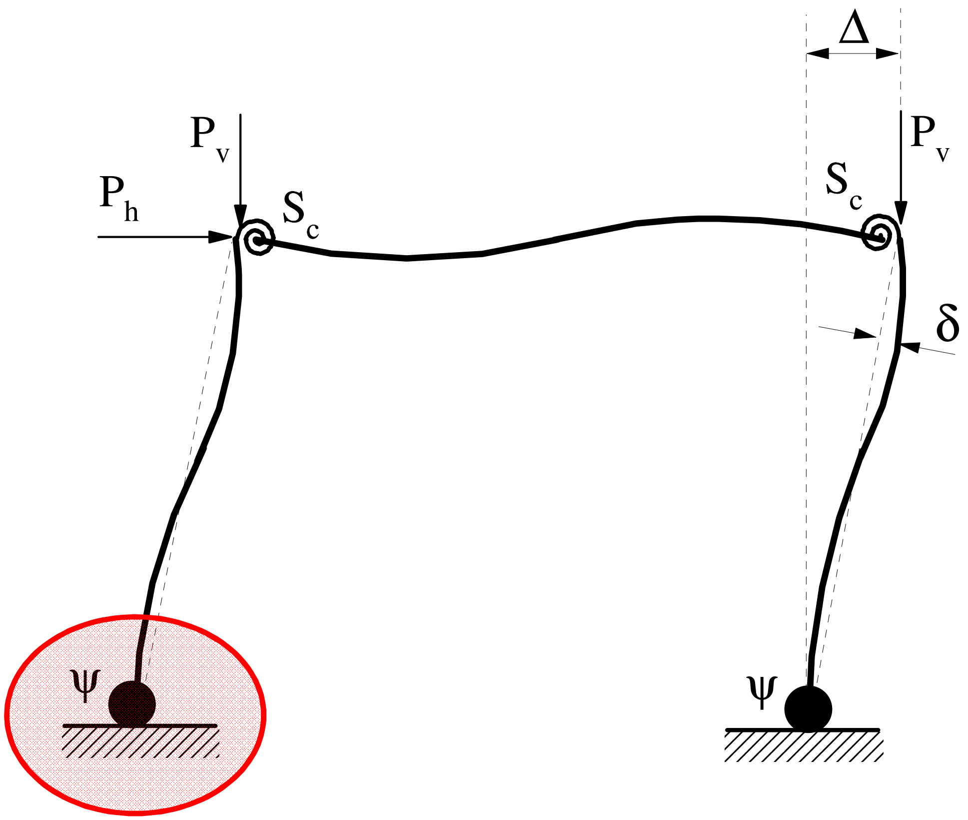
Semi-Rigid Connection

$$\mathbf{F}_{int}\left(\mathbf{U},\mathbf{P},\mathbf{S}_c\right)=\mathbf{F}_{ext}$$



Inelastic/Combined Effects

$$\mathbf{F}_{int}\left(\mathbf{U},\mathbf{P},\mathbf{S}_c,\psi\right)=\mathbf{F}_{ext}$$



CS-ASA

Computational System – Advanced Structural Analysis

Galvão (2000)

Gonçalves (2013)

Silva (2009)

Interactive graphics program (Prado, 2012)

Pre-processing

Definition of structural geometry, support conditions, applied loads, element properties and semi-rigid connections

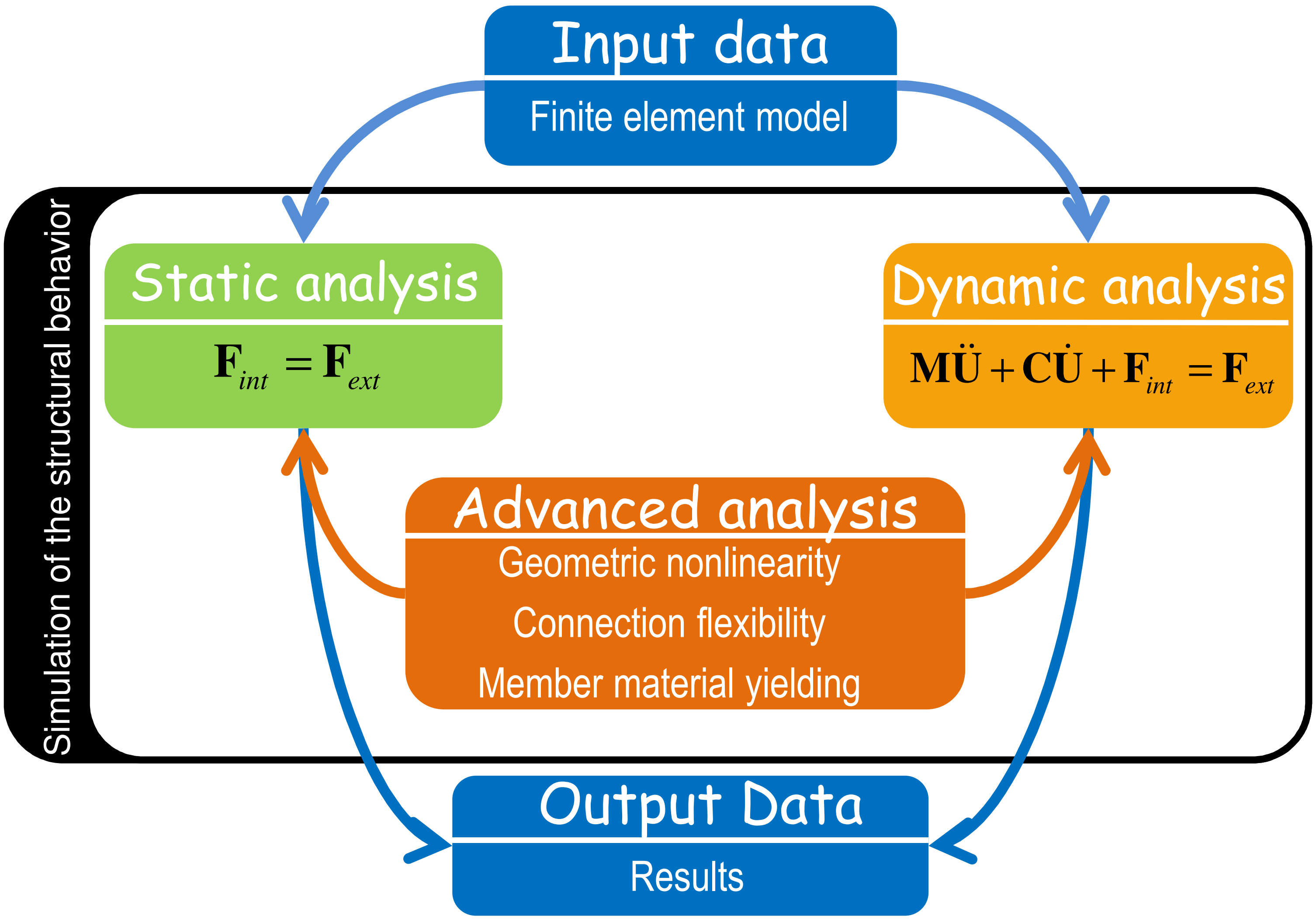
Analysis

First order elastic of 2D frames subjected to static and dynamic load

Second order elastic and inelastic analysis of 2D frames subjected to static and dynamic load

Pos-processing

Equilibrium paths, internal forces diagrams, degree of member section plastification, connection and section hysteretic behaviour, displacements, velocities and accelerations time histories


Input data
Finite element model
Simulation of the structural behavior
Static analysis
$\mathbf{F}_{int} = \mathbf{F}_{ext}$
Dynamic analysis
$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{F}_{int} = \mathbf{F}_{ext}$
Advanced analysis
Geometric nonlinearity
Connection flexibility
Member material yielding
Output Data
Results

Static Analysis

$$\mathbf{F}_{int} = \mathbf{F}_{ext}$$

FORMULATIONS

Geometric nonlinearity

SOF-1
SOF-2
SOF-3

$$\mathbf{F}_{int}\left(\mathbf{U},\mathbf{P}\right) = \mathbf{F}_{ext}$$

Connection flexibility

SRF-1
SRF-2
SRF-3

$$\mathbf{F}_{int}\left(\mathbf{U},\mathbf{P},\mathbf{S}_c\right) = \mathbf{F}_{ext}$$

Material yielding

PHF-1
PHF-2

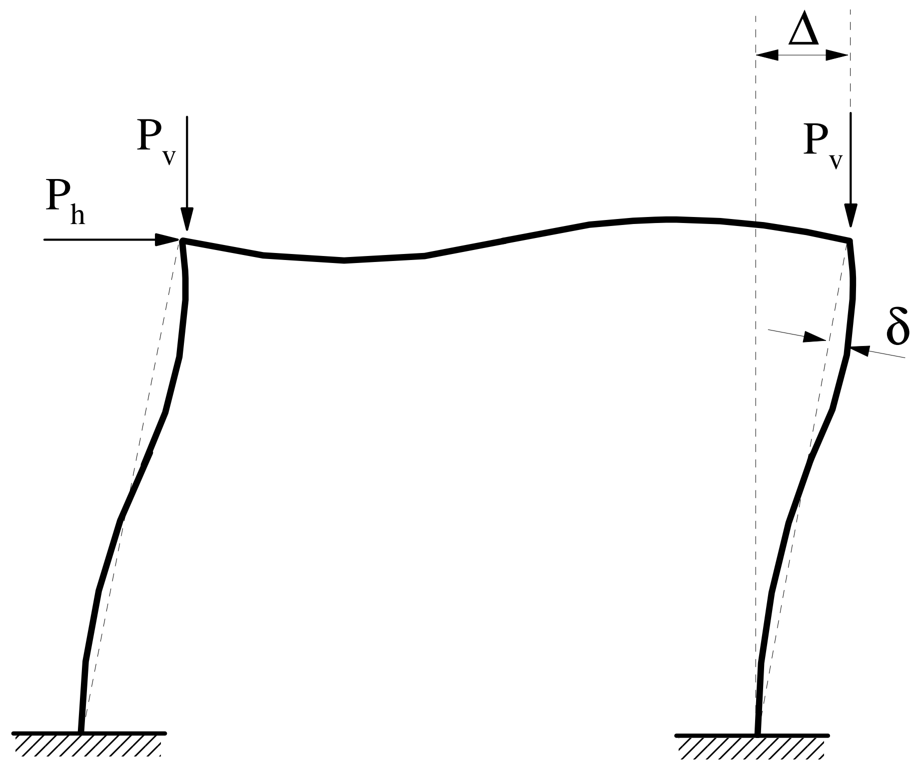
$$\mathbf{F}_{int}\left(\mathbf{U},\mathbf{P},\psi\right) = \mathbf{F}_{ext}$$

Combined effects

AAF-1
AAF-2

$$\mathbf{F}_{int}\left(\mathbf{U},\mathbf{P},\mathbf{S}_c,\psi\right) = \mathbf{F}_{ext}$$

Second-Order Effects



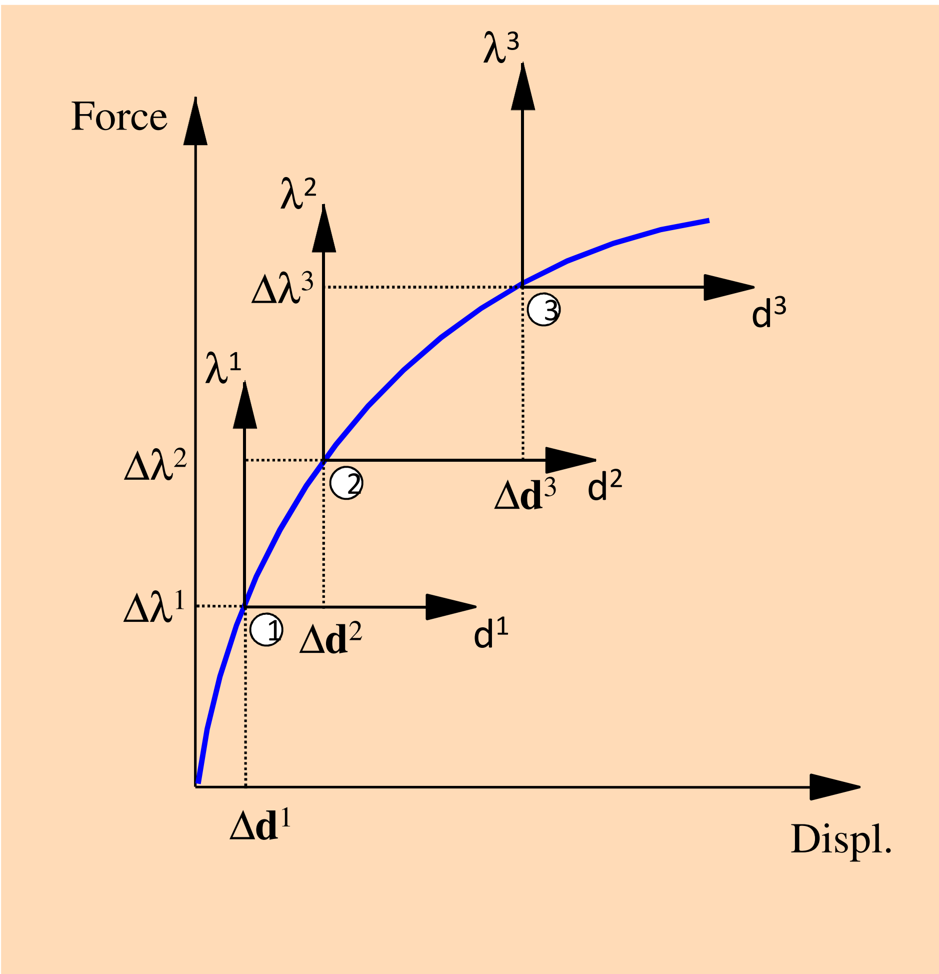
Basic Considerations...

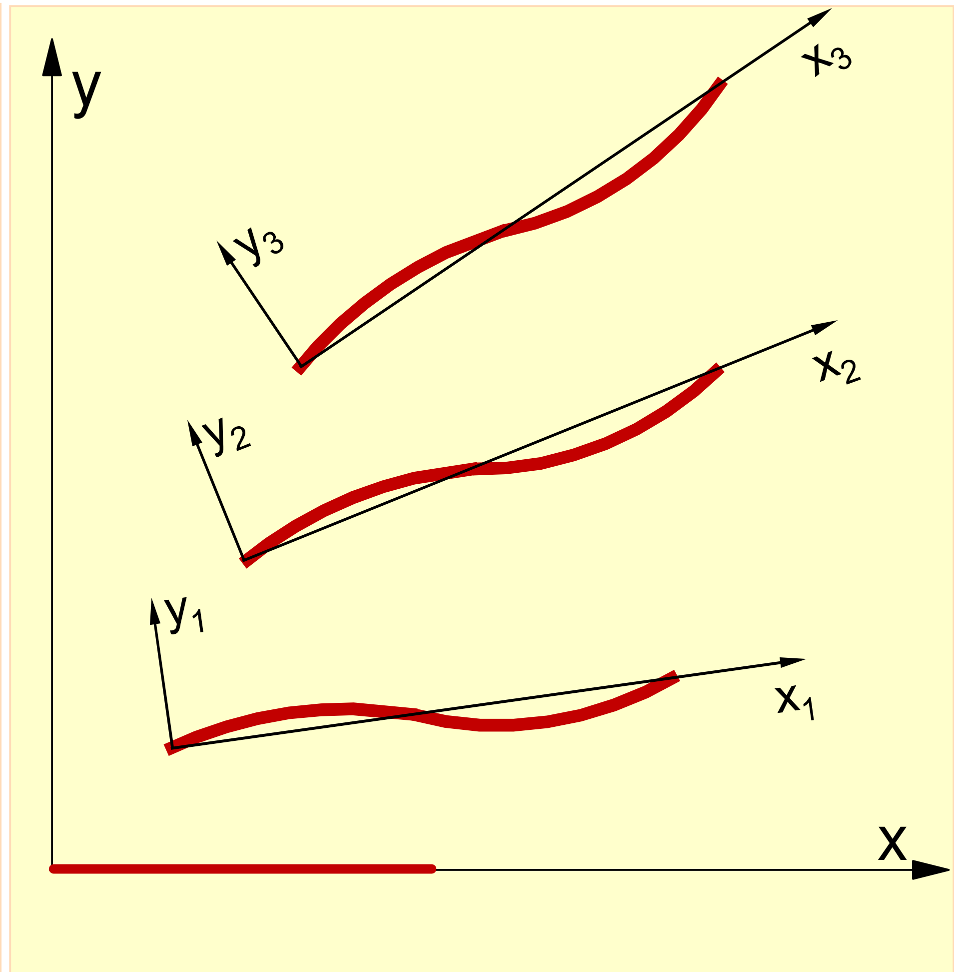
- The equilibrium must be satisfied on the deformed geometry of the structure
- Theory: Euler-Bernoulli or Timoshenko beam
- Kinematic relations
- Energy functional / Virtual work
- FE approximation: interpolation functions
- **Product: stiffness matrix and internal force vector**

Basic Considerations…

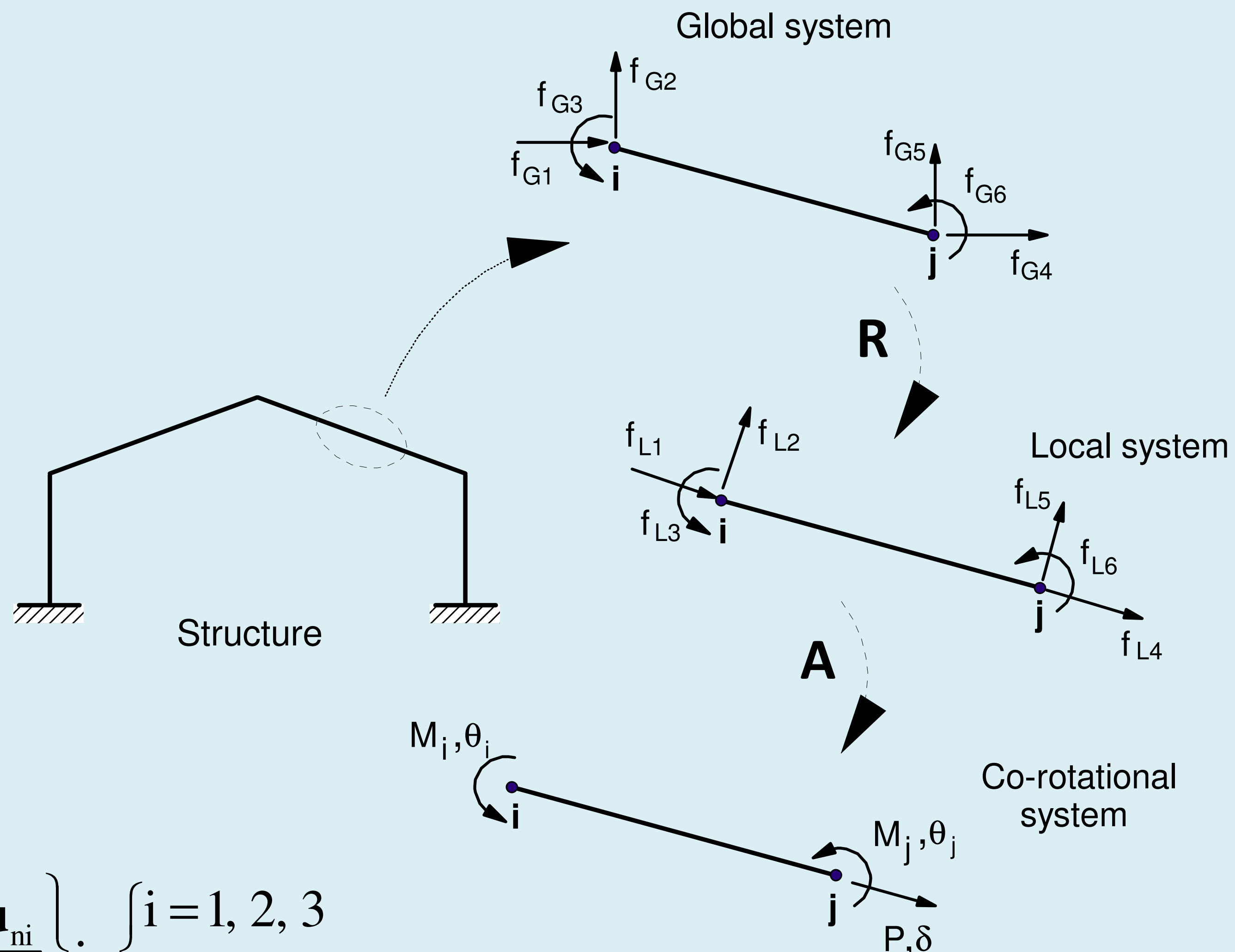
- **Type of referential: Updated L. or Total L.**
- **FE co-rotational formulation/system**

Updated Lagrangian Referential





Co-Rotational System



$$\mathbf{A} = \left\{ \frac{\partial \mathbf{u}_{ni}}{\partial \mathbf{u}_j} \right\}; \quad \begin{cases} i = 1, 2, 3 \\ j = 1, 2, 3, 4, 5, 6 \end{cases}$$

Pacoste and Eriksson (1997)

1. Kinematic Relations

$$\varepsilon_{xx} = \left(1+\frac{du}{dx}\right)\cos(\theta)+\left(\frac{dv}{dx}\right)\mathrm{sen}(\theta)$$

$$\gamma = \left(\frac{dv}{dx}\right)\cos(\theta)-\left(1+\frac{du}{dx}\right)\mathrm{sen}(\theta)$$

$$k = \frac{d\theta}{dx}$$

2. FE Approximations

$$u = H_1 u_i + H_2 u_j \qquad v = H_1 v_i + H_2 v_j$$

$$k = \frac{d\theta}{dx} = \frac{dH_1}{dx}\theta_i + \frac{dH_2}{dx}\theta_j$$

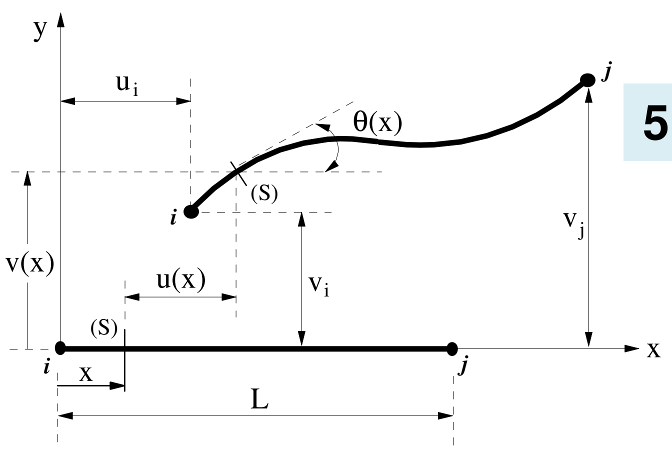
$$\text{with, } H_1 = 1 - \frac{x}{L} \qquad H_2 = \frac{x}{L}$$

3. Internal Deformation Energy

$$U = \frac{1}{2}\int_0^L \left[EA\varepsilon_{xx}^2 + GA\gamma^2 + EIk^2\right] dx$$

4. Internal Force Vector

$$fi_m = \frac{\partial U}{\partial u_m}$$

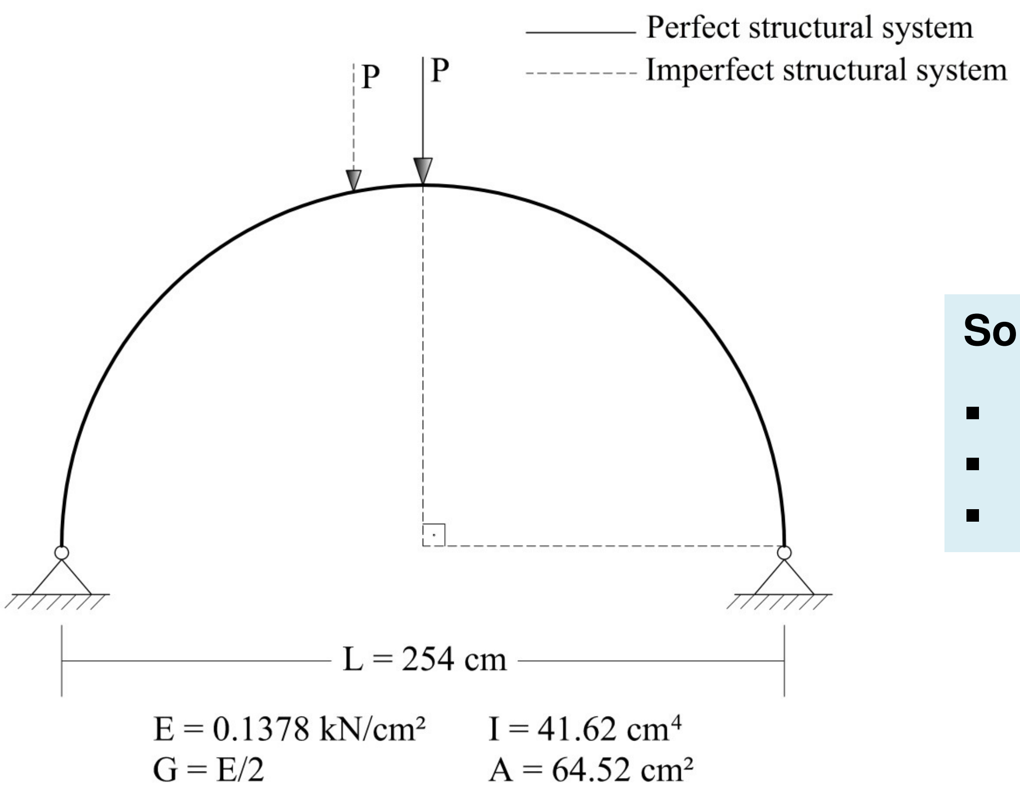


5. Stiffness Matrix

$$k_{mn} = \frac{\partial^2 U}{\partial u_m \partial u_n}$$

Circular Arch

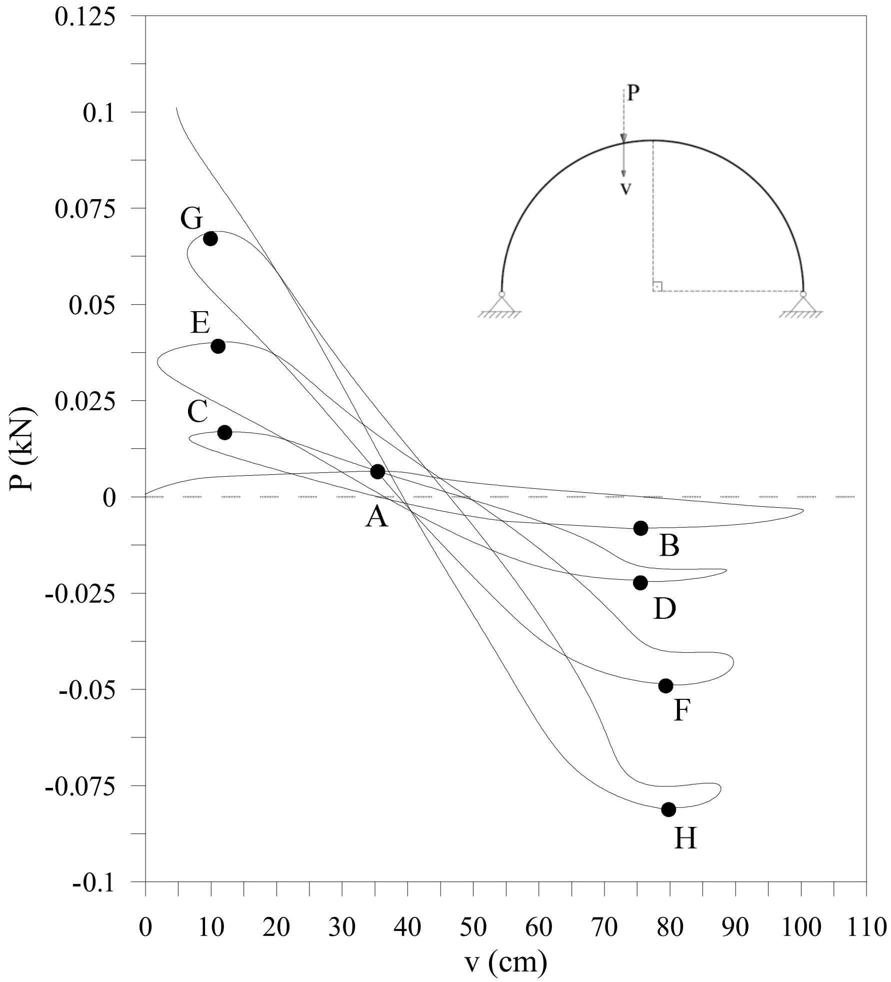
Yang and Kuo (1994)

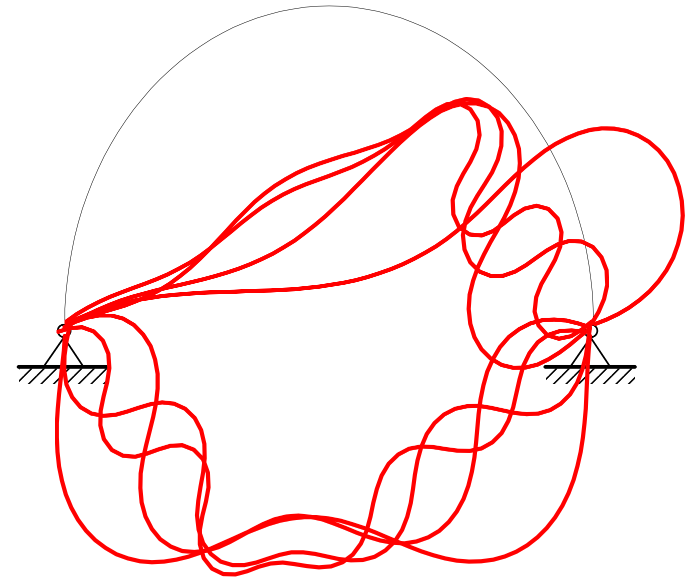


Solution Parameters:

- FEs : 68
- P ($\Delta\lambda^0$): 0.4 N
- Full Newton-Raphson

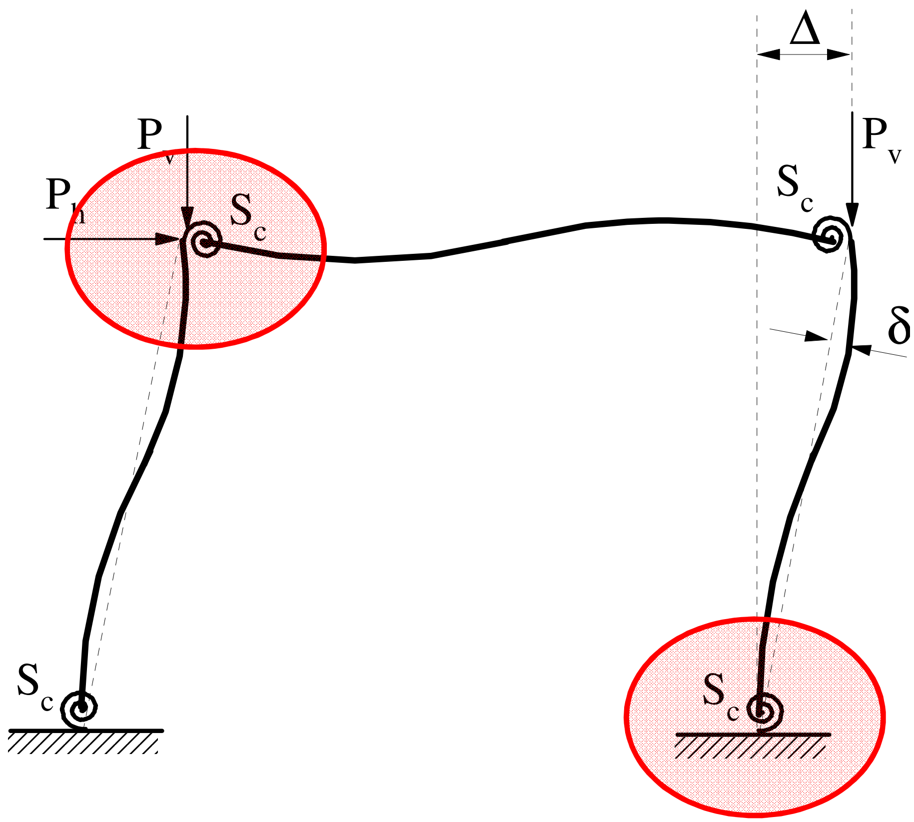
Results: Imperfect System





L.P. Load	CS-ASA	Yang e Kuo (1994)
A	5.904	5.813
B	-8.463	-8.498
C	16.524	16.149
D	-21.901	-22.162
E	39.521	38.566
F	-49.712	-49.896
G	67.822	64.875
H	-81.958	-82.420

Semi-Rigid Connections



Basic Considerations…

- **Affects strength and stiffness of the steel frame**
- **Behavior under monotonic loading**
- **Mathematical models**
- **Semi-rigid FE formulations (3 in CS-ASA)**
- **Numerical example**

The connections:

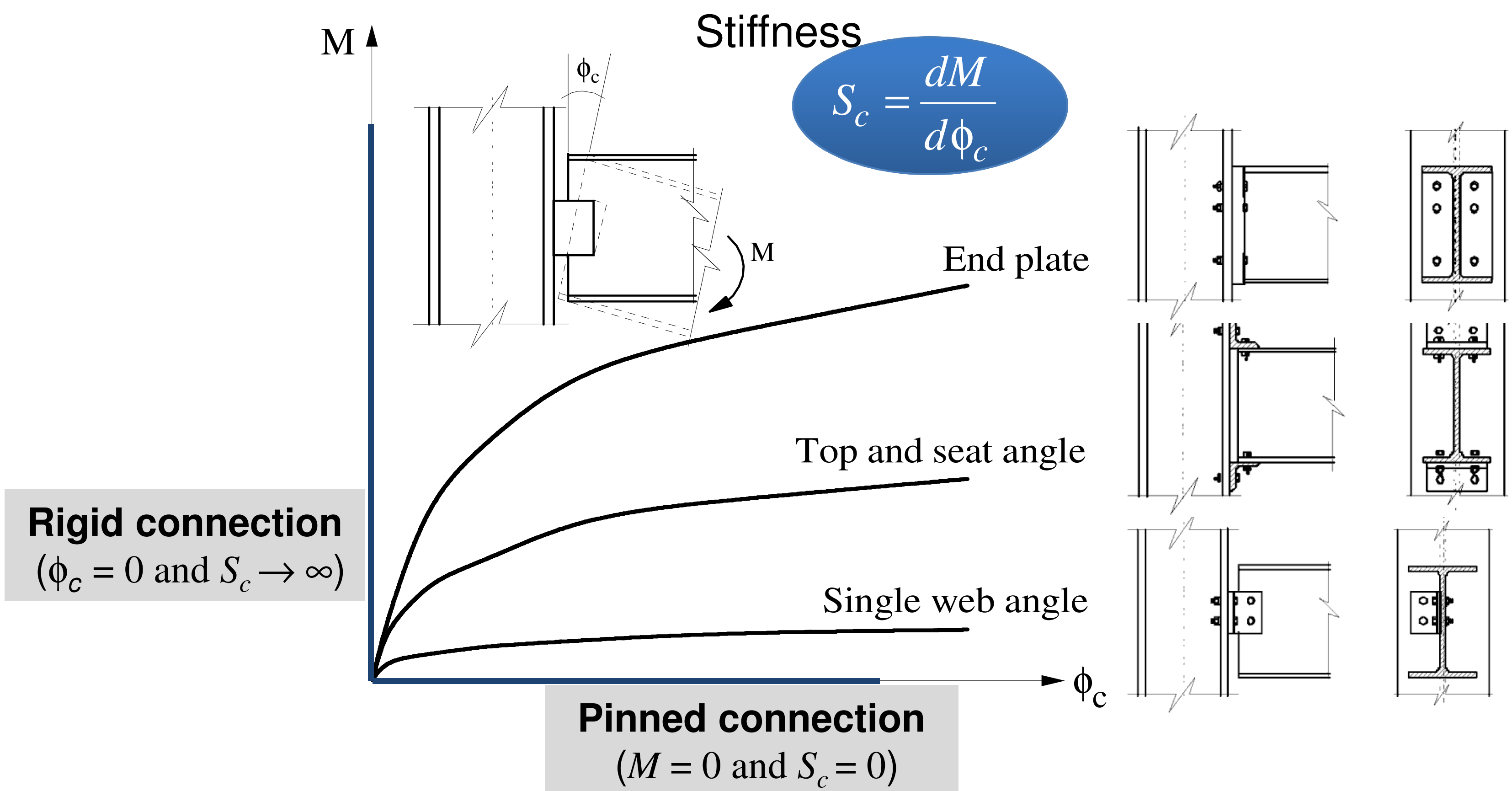
- Transfer of forces and moments
- Global stability and strength of the structural system and internal forces distribution

Stiffness criteria: rigid, semi-rigid or pinned

Behavior of semi-rigid connection:

- Moment-rotation curve M-ϕ_c (experiments, anal. models)
- Mathematical models: monotonic and cyclic loads

Behavior under Monotonic Loading



Mathematical Models

$$M = f(\phi_c)$$

Linear $M = S_{cini}\phi_c$

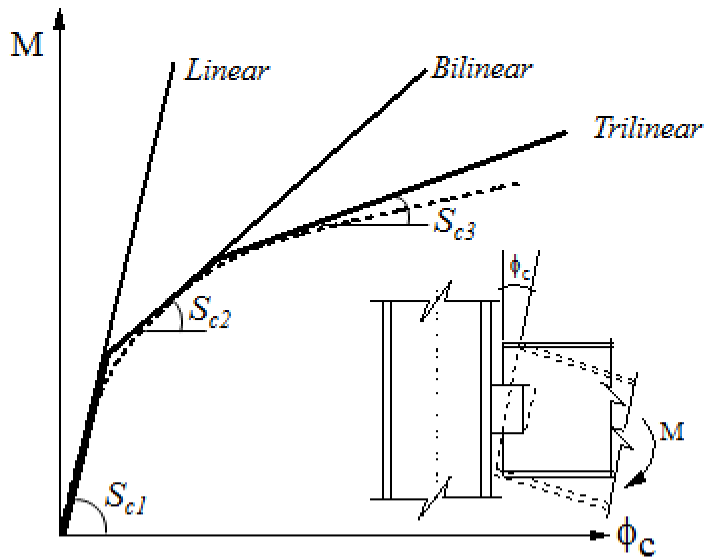
Exponencial
(Chen e Lui, 1986)

$$M = M_o + \sum_{m=1}^{n} C_j \left[1 - exp\left(\frac{-|\phi_c|}{2m\alpha} \right) \right] + R_p |\phi_c|$$

Power
(Richard and Abbott, 1975)

$$- + R_p |\phi_c|$$

Multilinear

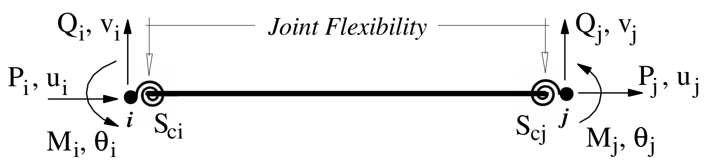


Semi-Rigid FE Formulations

CS-ASA: Connection flexibility

- SRF-1 Chan and Chui (2000)
- SRF-2 Chen and Lui (1997)
- SRF-3 Sekulovic and Salatic (2001; eccentric connection)

Semi-rigid connection: **Spring Element**

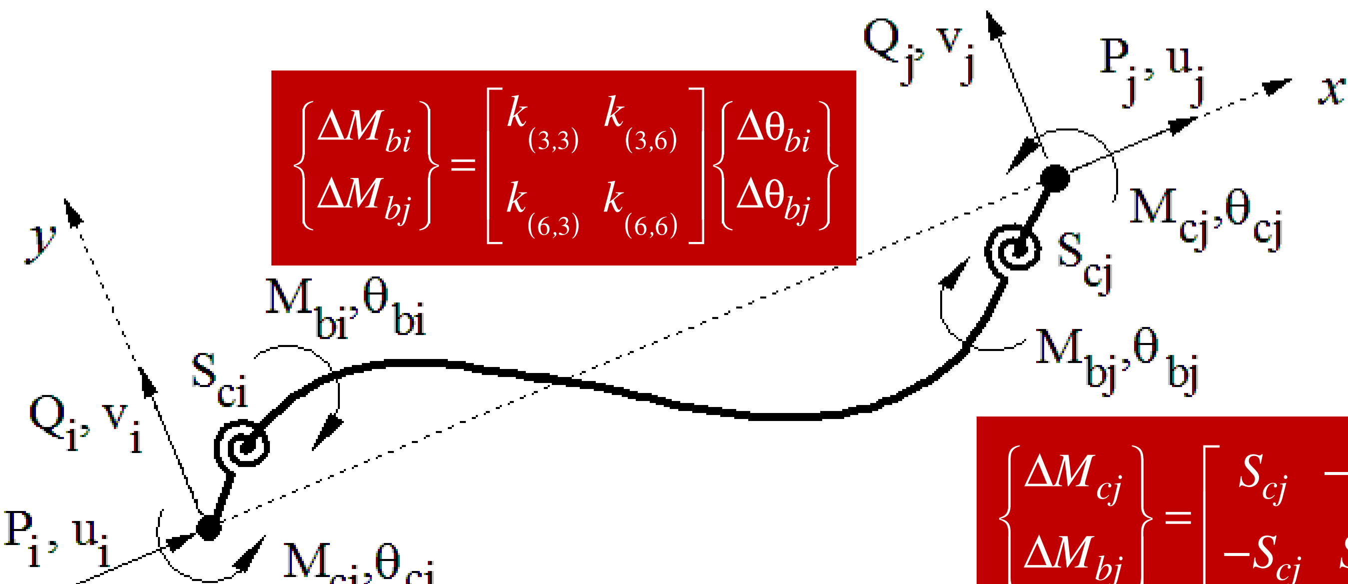


$$\mathbf{K} = f\left(\mathbf{U}, \mathbf{P}, \mathbf{S}_c\right)$$

Semi-Rigid FE Formulations

Chan and Chui (2000)

$$\begin{Bmatrix} \Delta M_{bi} \\ \Delta M_{bj} \end{Bmatrix} = \begin{bmatrix} k_{(3,3)} & k_{(3,6)} \\ k_{(6,3)} & k_{(6,6)} \end{bmatrix} \begin{Bmatrix} \Delta\theta_{bi} \\ \Delta\theta_{bj} \end{Bmatrix}$$



Node j:

$$\begin{Bmatrix} \Delta M_{cj} \\ \Delta M_{bj} \end{Bmatrix} = \begin{bmatrix} S_{cj} & -S_{cj} \\ -S_{cj} & S_{cj} \end{bmatrix} \begin{Bmatrix} \Delta\theta_{cj} \\ \Delta\theta_{bj} \end{Bmatrix}$$

Node i:

$$\begin{Bmatrix} \Delta M_{ci} \\ \Delta M_{bi} \end{Bmatrix} = \begin{bmatrix} S_{ci} & -S_{ci} \\ -S_{ci} & S_{ci} \end{bmatrix} \begin{Bmatrix} \Delta\theta_{ci} \\ \Delta\theta_{bi} \end{Bmatrix}$$

Semi-Rigid FE Formulations

$$\begin{Bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta M_i \\ \Delta P_j \\ \Delta Q_j \\ \Delta M_j \end{Bmatrix} = \begin{bmatrix} k_{(1,1)} & k_{(1,2)} & k_{(1,3)} & k_{(1,4)} & k_{(1,5)} & k_{(1,6)} \\ k_{(1,2)} & k^*_{(2,2)} & k^*_{(2,3)} & k_{(2,4)} & k^*_{(2,5)} & k^*_{(2,6)} \\ k_{(3,1)} & k^*_{(3,2)} & k^*_{(3,3)} & k_{(3,4)} & k^*_{(3,5)} & k^*_{(3,6)} \\ k_{(4,1)} & k_{(4,2)} & k_{(4,3)} & k_{(4,4)} & k_{(4,5)} & k_{(4,6)} \\ k_{(5,1)} & k^*_{(5,2)} & k^*_{(5,3)} & k_{(5,4)} & k^*_{(5,5)} & k^*_{(5,6)} \\ k_{(6,1)} & k^*_{(6,2)} & k^*_{(6,3)} & k_{(6,4)} & k^*_{(6,5)} & k^*_{(6,6)} \end{bmatrix} \begin{Bmatrix} \Delta u_i \\ \Delta v_i \\ \Delta \theta_i \\ \Delta u_j \\ \Delta v_j \\ \Delta \theta_j \end{Bmatrix}$$

$$\mathbf{K}^* = \begin{bmatrix} 1/L & 1/L \\ 1 & 0 \\ -1/L & -1/L \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} S_{ci} & 0 \\ 0 & S_{cj} \end{bmatrix} - \frac{1}{\beta} \begin{bmatrix} S_{ci} & 0 \\ 0 & S_{cj} \end{bmatrix} \begin{bmatrix} S_{cj} + k_{(6,6)} & k_{(3,6)} \\ k_{(6,3)} & S_{ci} + k_{(3,3)} \end{bmatrix} \begin{bmatrix} S_{ci} & 0 \\ 0 & S_{cj} \end{bmatrix} \right) \begin{bmatrix} 1 & 1/L & 0 & -1/L \\ 0 & 1/L & 1 & -1/L \end{bmatrix}$$

Numerical Example

Vogel Frame

Chan and Chui (2000)

Analysis

$F_1 = 10.23\lambda$ kN

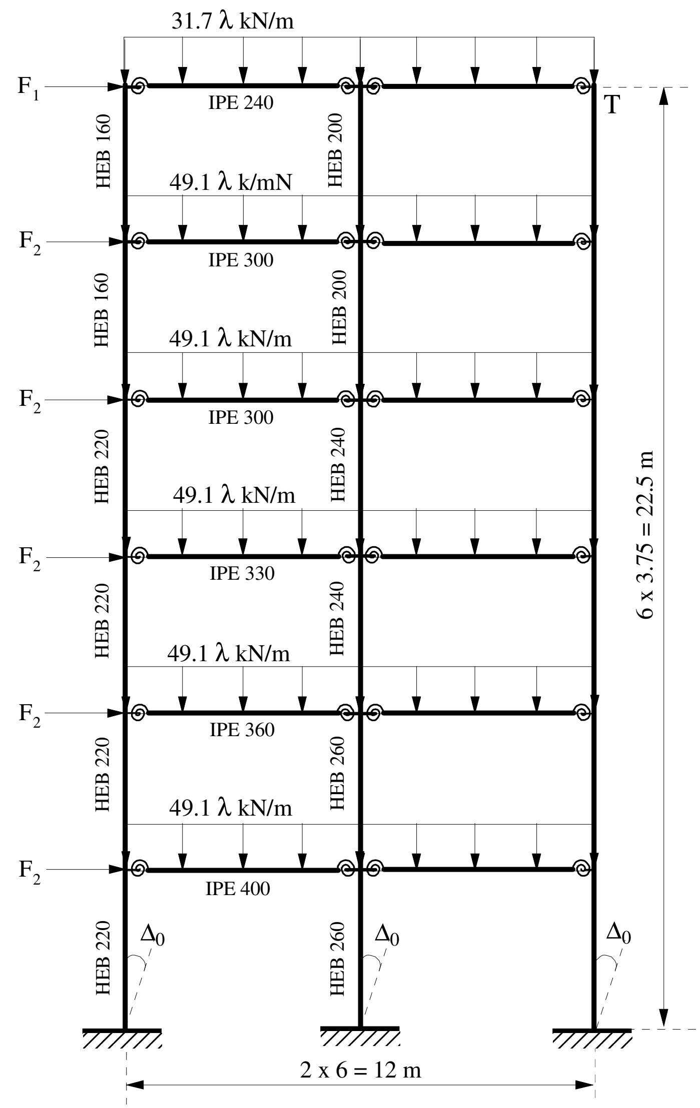
$F_2 = 20.44\lambda$ kN

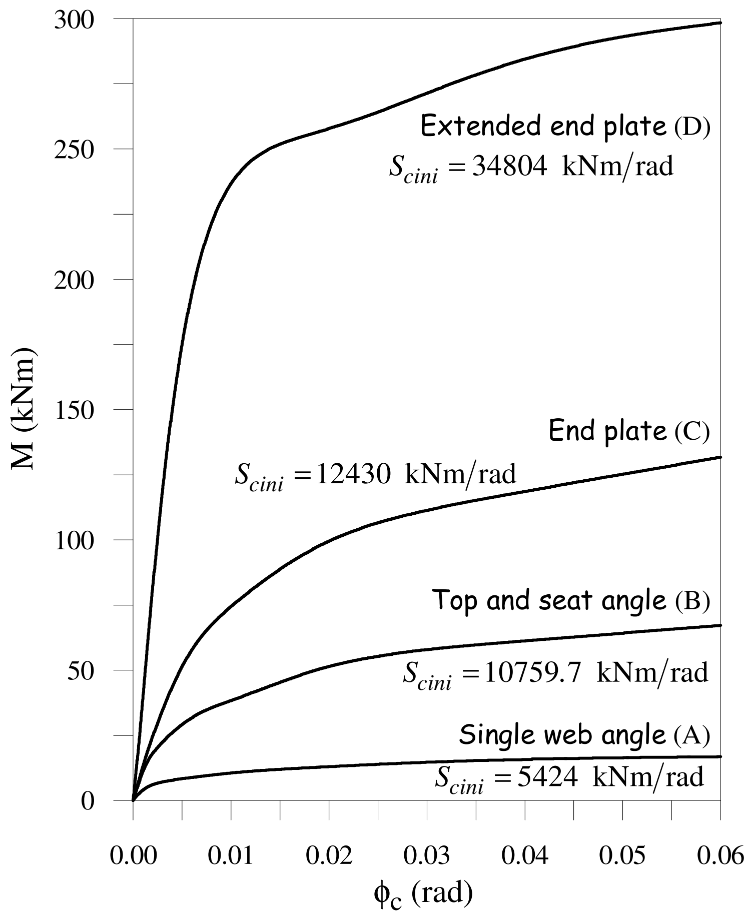
$\Delta_0 = 1/300$

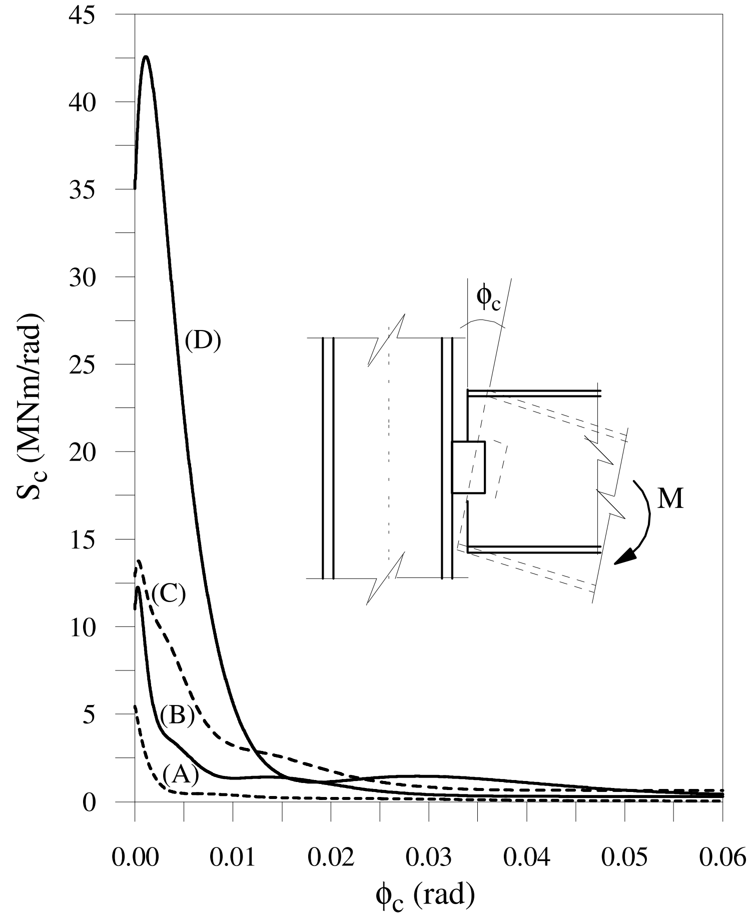
$E = 205$ GPa

Beam-column joints

- Rigid
- Single web angle (A)
- Top and seat angle (B)
- End-plate (C)
- Extended end-plate (D)



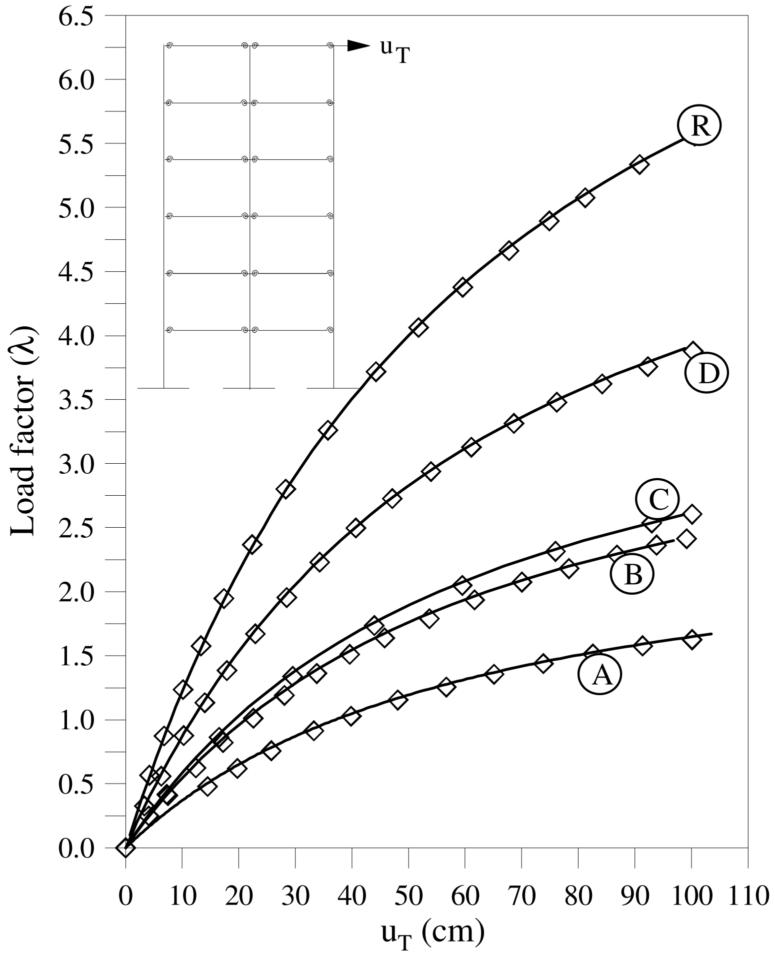




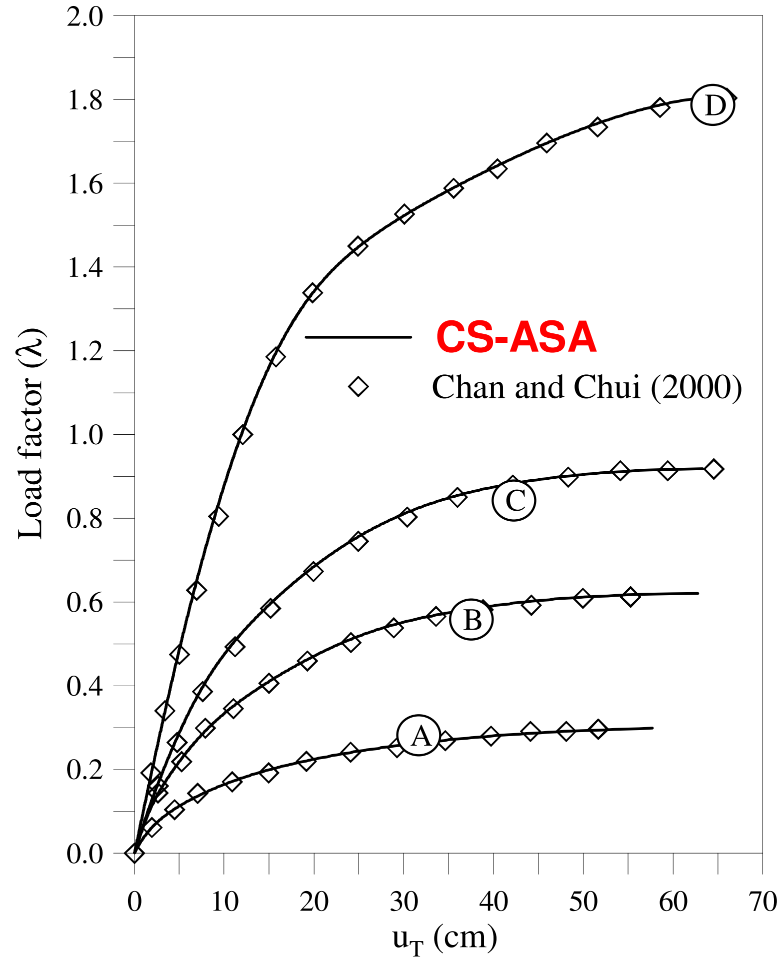
Exponential Model

$$M = M_o + \sum_{m=1}^{n} C_j \left[1 - exp\left(\frac{-|\phi_c|}{2m\alpha} \right) \right] + R_{kf} \, |\phi_c|$$

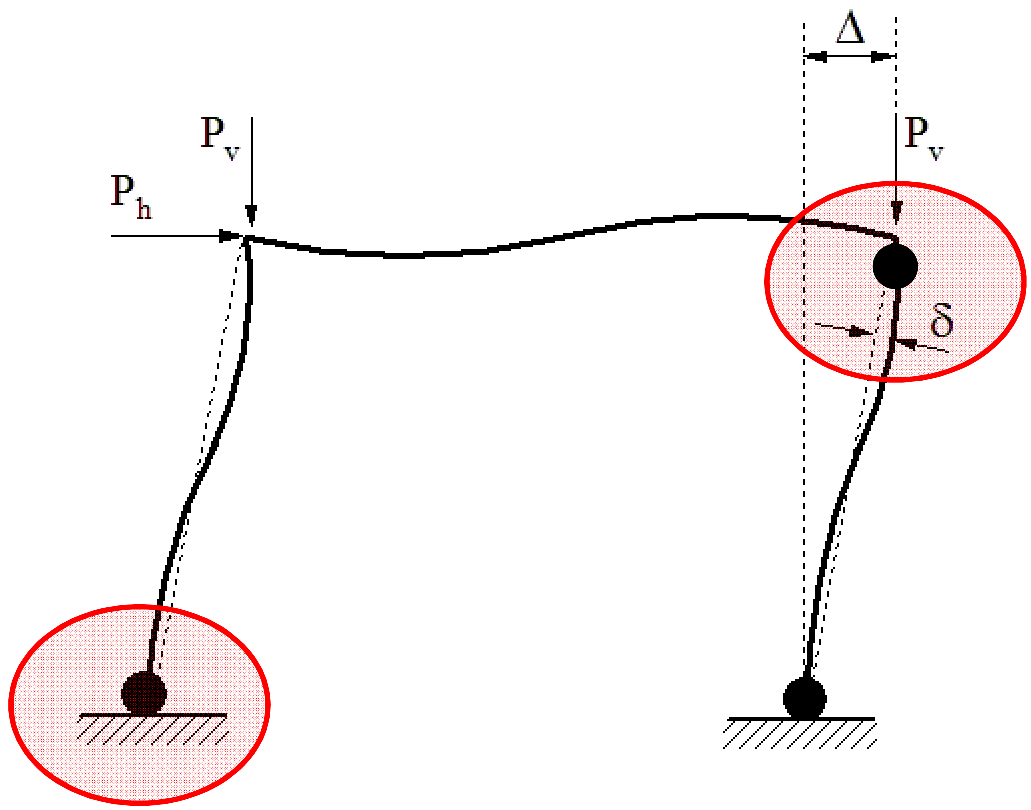
Linear M-ϕ_c behavior



Nonlinear M-ϕ_c behavior



Inelastic Effects

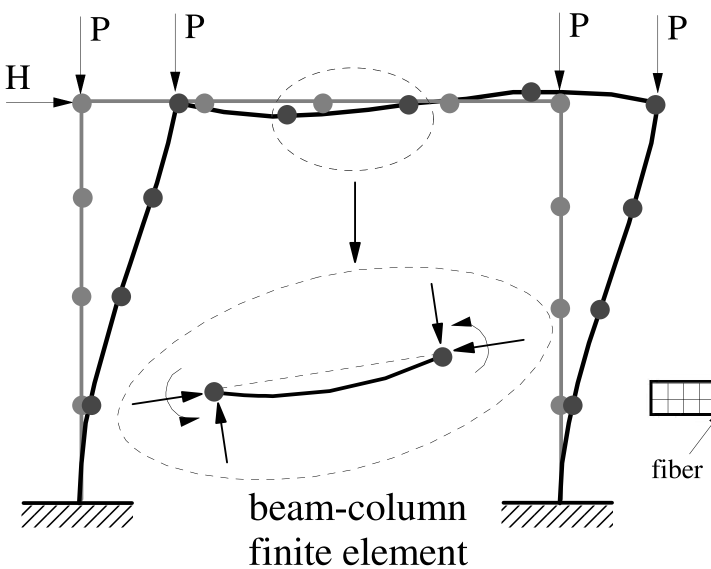


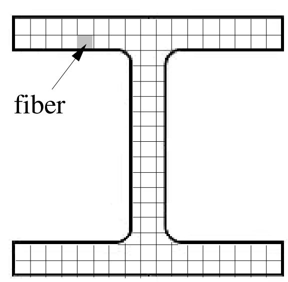
Basic Considerations...

- Affects strength and stiffness of the steel frame
- Methodologies of inelastic analysis
- Generalized inelastic formulation
- Numerical example

Metodologies of Inelastic Analysis

Plastic-zone method





Meshed section with fibers

Stress state of each fiber determined

Gradual spread of yielding

Second-order effect

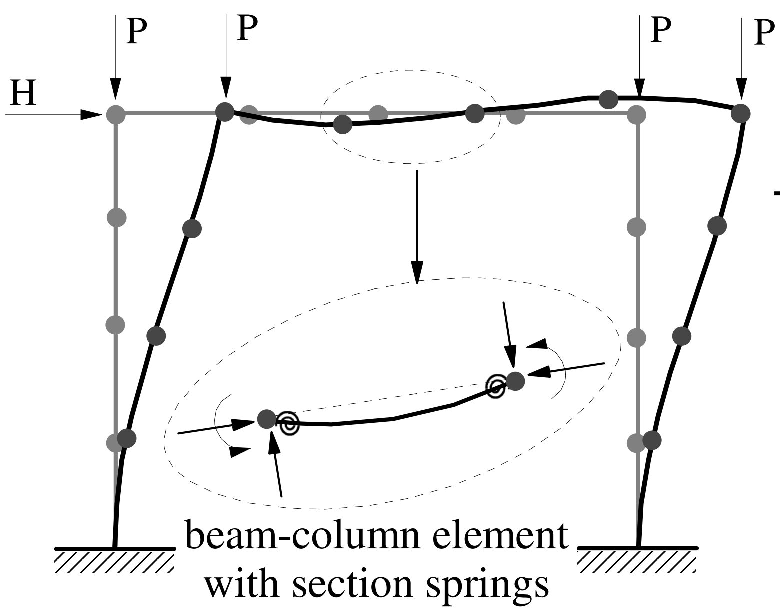
Residual stresses

Solution is considered “exact”

High computational cost

Metodologies of Inelastic Analysis

Plastic-hinge method



Material yielding effect: plastic-hinge at the member end

The portion within the element is assumed to behave elastically

Elasto-plastic hinge method

Refined-plastic hinge method

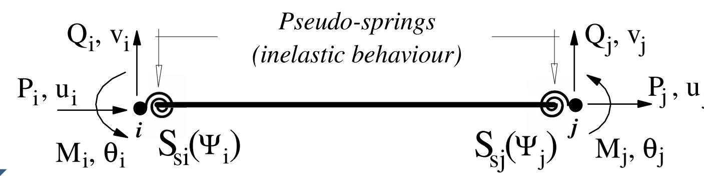
Section gradual yielding
Residual stresses
Initial geometric imperfections

Generalized Inelastic Formulation

CS-ASA:

- **PHF-1** Liew et al. (1993; AISC and NBR)
- **PHF-2** Chan and Chui (2000; BS5950)

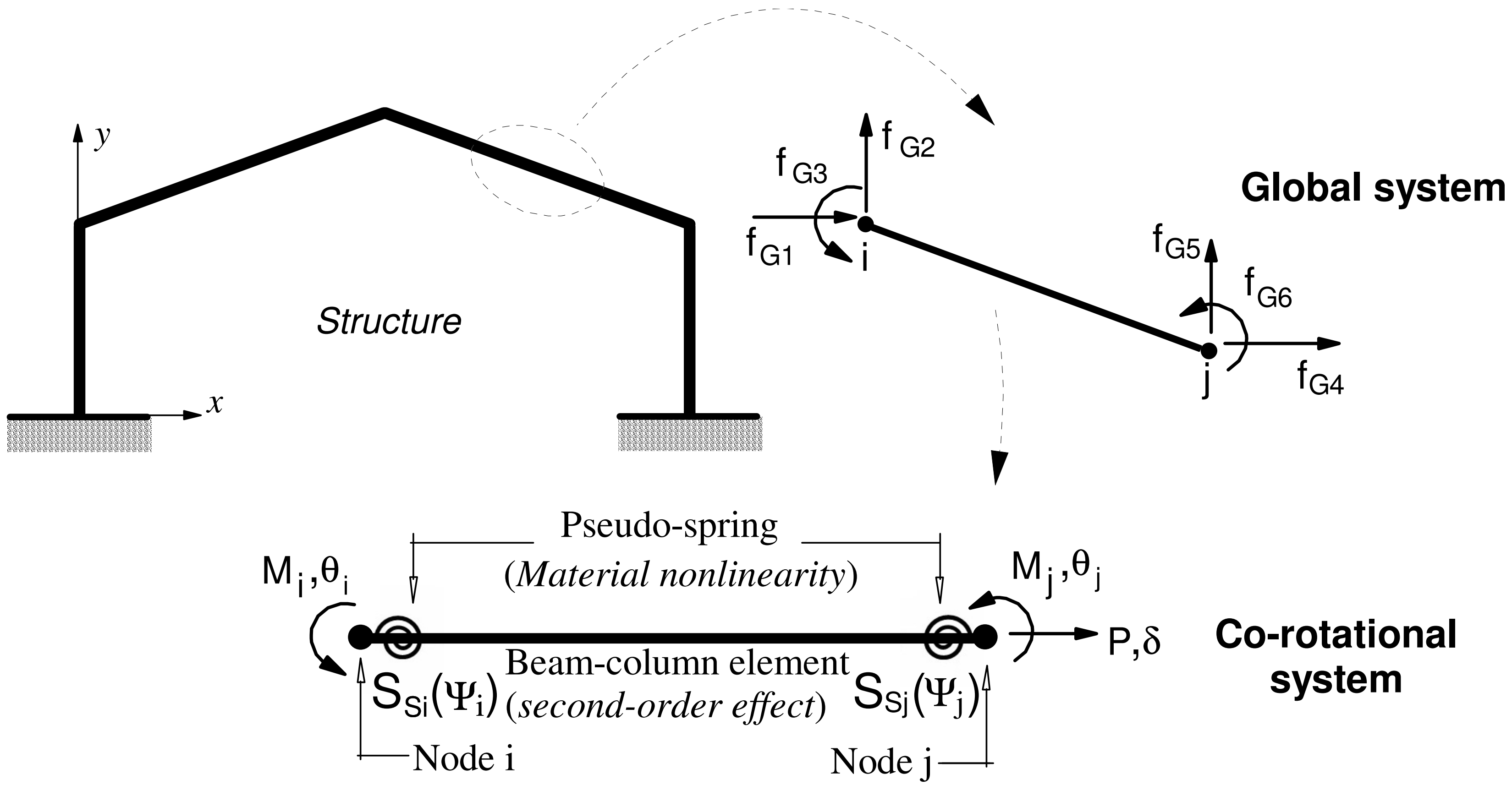
Inelastic effects: **Pseudo-Spring Element**



$$\mathbf{K} = f\left(\mathbf{U}, \mathbf{P}, \mathbf{\Psi}\right) \qquad S_s = \frac{6EI}{L}\frac{\psi}{1-\psi}$$

Generalized Inelastic Formulation

The Finite Element



Generalized Inelastic Formulation

Force-Displacement Relation

$$\Delta \mathbf{f}_C = \mathbf{K}_C \Delta \mathbf{u}_C$$

$$\begin{Bmatrix} \Delta P \\ \Delta M_i \\ \Delta M_j \end{Bmatrix} = \begin{bmatrix} C_1 A/L & 0 & 0 \\ 0 & C_2\left[k_{ii}K_{jj} - k_{ij}k_{ji}\left(1 - C_3\right)\right] & C_6 k_{ij} \\ 0 & C_4\left[k_{ii}K_{jj} - k_{ij}k_{ji}\left(1 - C_5\right)\right] & C_6 k_{ji} \end{bmatrix} \begin{Bmatrix} \Delta \delta \\ \Delta \theta_i \\ \Delta \theta_j \end{Bmatrix}$$

k_{ii}, k_{ij}, k_{ji} and k_{jj}: Geometric nonlinear formulation

$C_{m\ (m=1,..,6)}$: Liew or Chan-Chui formulations

Generalized Inelastic Formulation

Terms in the Equilibrium Equation

Formulation	Parameters					
	C_1	C_2	C_3	C_4	C_5	C_6
Liew	E_t	ψ_i/k_{ji}	ψ_j	ψ_j/k_{ij}	ψ_i	$\psi_i\psi_j$
Chan-Chui	E	$\dfrac{\psi_i}{\beta_1(1-\psi_i)}$	$\dfrac{\psi_j}{(1-\psi_j)}\beta_2 k_{ii}$	$\dfrac{\psi_j}{\beta_1(1-\psi_j)}$	$\dfrac{\psi_i}{(1-\psi_i)}\beta_2 k_{jj}$	$\dfrac{6EI\psi_i\psi_j}{L(1-\psi_i)(1-\psi_j)}$

$$\beta_1 = \frac{L}{6EI}\left[\left(\frac{6EI}{L}\frac{\psi_j}{(1-\psi_j)}+k_{ii}\right)\left(\frac{6EI}{L}\frac{\psi_i}{(1-\psi_i)}+k_{jj}\right)-k_{ij}k_{ji}\right];\quad \beta_2 = \frac{6EI}{L}\frac{1}{k_{ij}k_{ji}}$$

I: moment of inertia; L: element length

E: Young's modulus; E_t: tangent module (AISC-LRFD, CRC)

ψ: strength reduction parameter (gradual cross-section yielding at the member end)

Generalized Inelastic Formulation

Strength Reduction Parameter ψ

Liew Formulation

$$\boxed{\psi = 4\alpha(1-\alpha)}$$

$\alpha \leq 0.5 \Rightarrow \psi = 1$ Elastic state

$\alpha = 1.0 \Rightarrow \psi = 0$ Full plastic state

α: force state parameter

Chan-Chui Formulation

$$\boxed{\psi = \frac{|M_{pr} - M|}{|M_{pr} - M| + |M - M_{er}|}}$$

$M \leq M_{er} \Rightarrow \psi = 1$ Elastic state

$M \geq M_{pr} \Rightarrow \psi = 0$ Full plastic state

M_{pr}: reduced moment plastic

M_{er}: reduced first yield moment

Generalized Inelastic Formulation

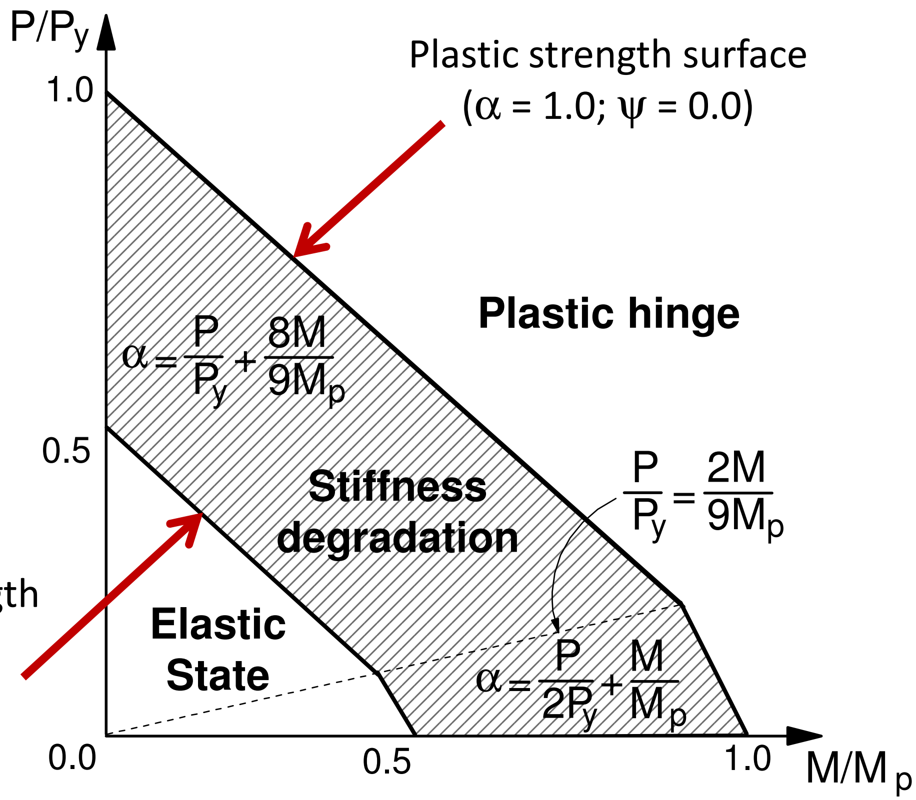
Plastic Strength Surface

Liew Formulation

AISC-LRFD (1999, 2005)
NBR 8800 (2008)

$$\alpha = \frac{P}{2P_y} + \frac{M}{M_p}, \text{ for } \frac{P}{P_y} < \frac{2}{9}\frac{M}{M_p}$$

$$\alpha = \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p}, \text{ for } \frac{P}{P_y} \geq \frac{2}{9}\frac{M}{M_p}$$



Initial yielding strength surface
($\alpha = 0.5$; $\psi = 0.0$)

Generalized Inelastic Formulation

Plastic Strength Surface

Chan-Chui Formulation

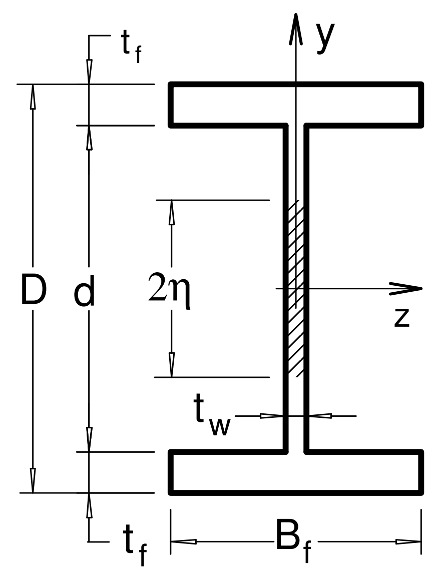
BS5950 (1990)

$$M_{pr} = \left[B_f\, t_f\,(D - t_f) + \left((d/2)^2 - \eta^2 \right) t_w \right] \sigma_y, \text{ for } \eta \le d/2$$

$$\eta = \frac{P}{2\sigma_y t_w}, \text{ for } \eta \le d/2$$

$$M_{pr} = \left[(d/2)^2 - \eta^2 \right] B_f \sigma_y, \text{ for } d/2 < \eta \le d/2 + t_f$$

$$\eta = \frac{(P - \sigma_y t_w d)}{2B_f\sigma_y}, \text{ for } d/2 < \eta \le d/2 + t_f$$



Cross-section

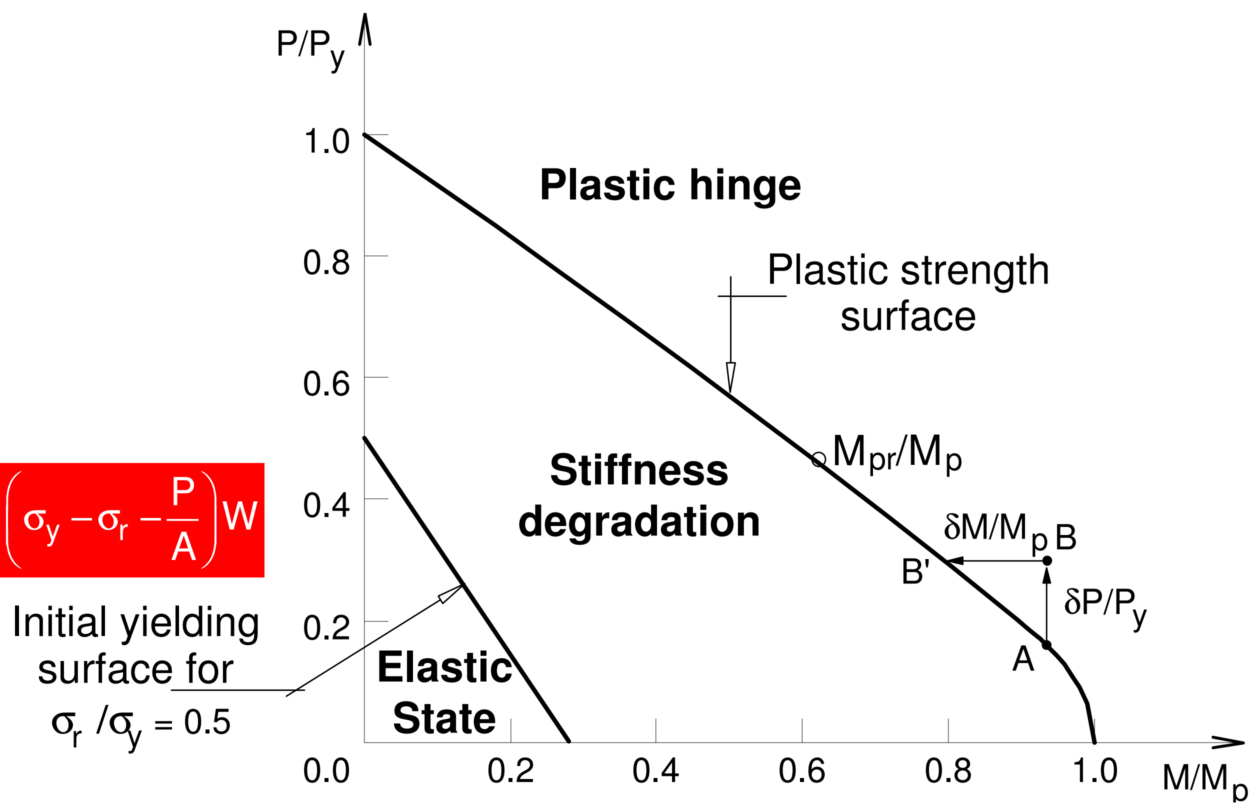
Generalized Inelastic Formulation

Plastic Strength Surface

Chan-Chui Formulation

Plastic strength surface for section HEB 220

$$M_{er} = \left(\sigma_y - \sigma_r - \frac{P}{A}\right)W$$



Generalized Inelastic Formulation

Load Vector Correction

$$\Delta \mathbf{f}_c = \mathbf{K}_{ch} \Delta \mathbf{u}_c + \Delta \mathbf{f}_{ps}$$

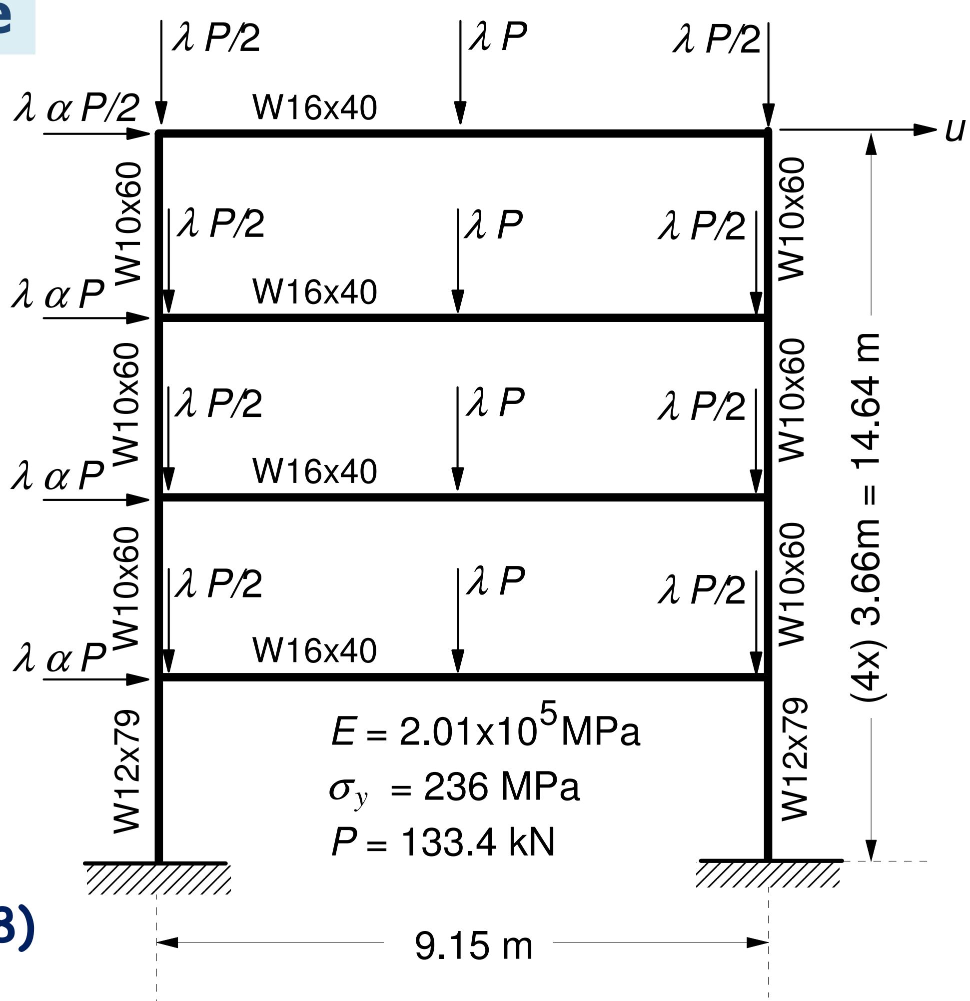
$$\begin{Bmatrix} \Delta P \\ \Delta M_i \\ \Delta M_j \end{Bmatrix} = \begin{bmatrix} C_1 A/L & 0 & 0 \\ 0 & C_7 K_{22} & 0 \\ 0 & 0 & C_8 K_{33} \end{bmatrix} \begin{Bmatrix} \Delta \delta \\ \Delta \theta_i \\ \Delta \theta_j \end{Bmatrix} + \begin{Bmatrix} 0 \\ \zeta_1 \\ \zeta_2 \end{Bmatrix}$$

Plastic hinge	**Parameters**			
	C_7	C_8	ζ_1	ζ_2
End i	0	1	δM_{pr_i}	$\delta M_{pr_i} \left(k_{c(3,2)} / K_{c(2,2)} \right)$
End j	1	0	$\delta M_{pr_j} \left(k_{c(2,3)} / K_{c(3,3)} \right)$	δM_{pr_j}
Ends i and j	0	0	δM_{pr_i}	δM_{pr_j}

$$K_{22} = k_{c(2,2)} - k_{c(2,3)} k_{c(3,2)} / k_{c(3,2)} \, ; \; K_{33} = k_{c(3,3)} - k_{c(2,3)} k_{c(3,2)} / k_{c(2,2)}$$

Numerical Example

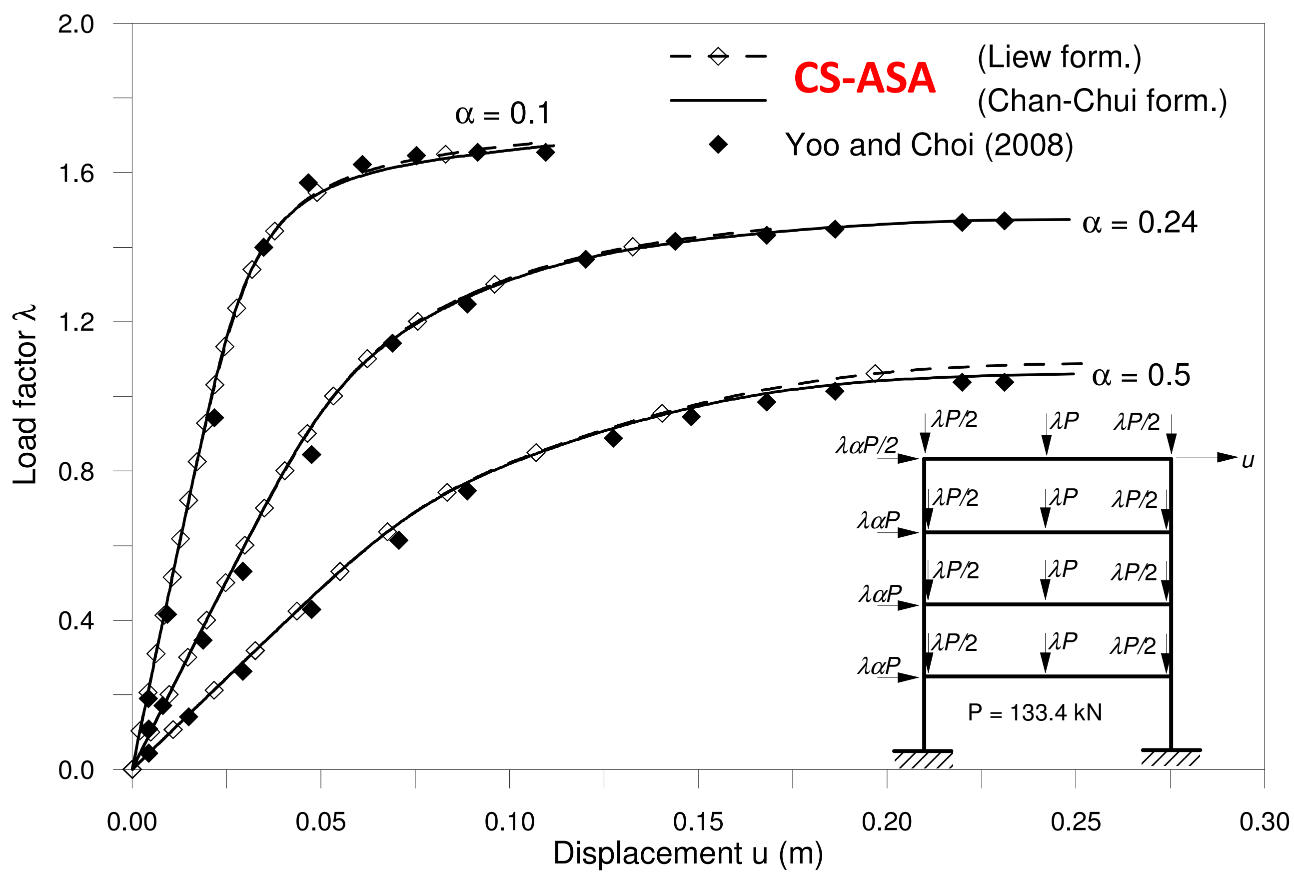
Four-story frame



Yoo and Choi (2008)

Numerical Example

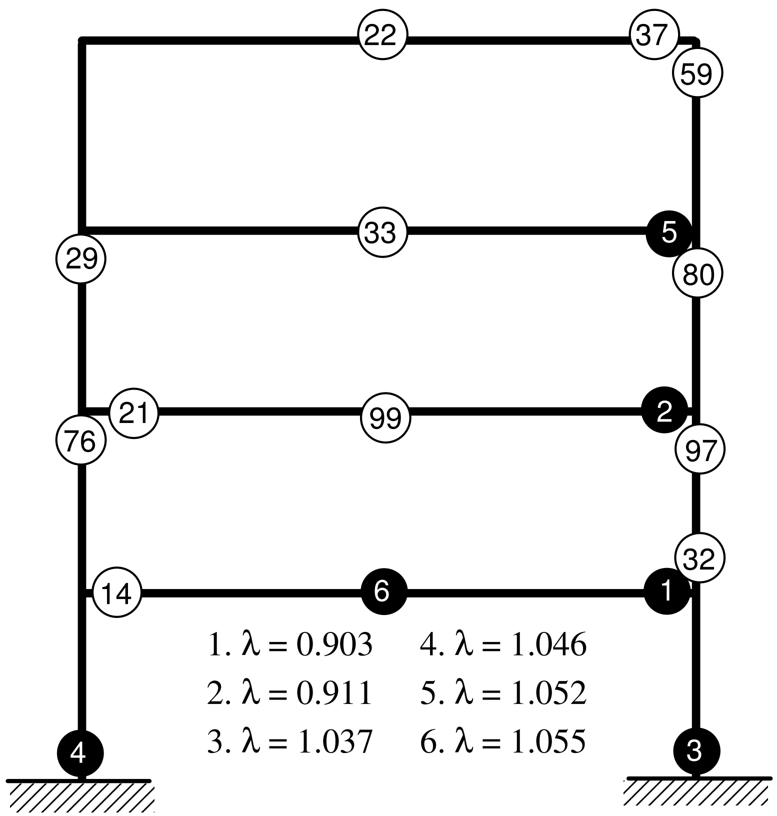
Load-Deflection Curves



Plastic-hinge Formation

CS-ASA

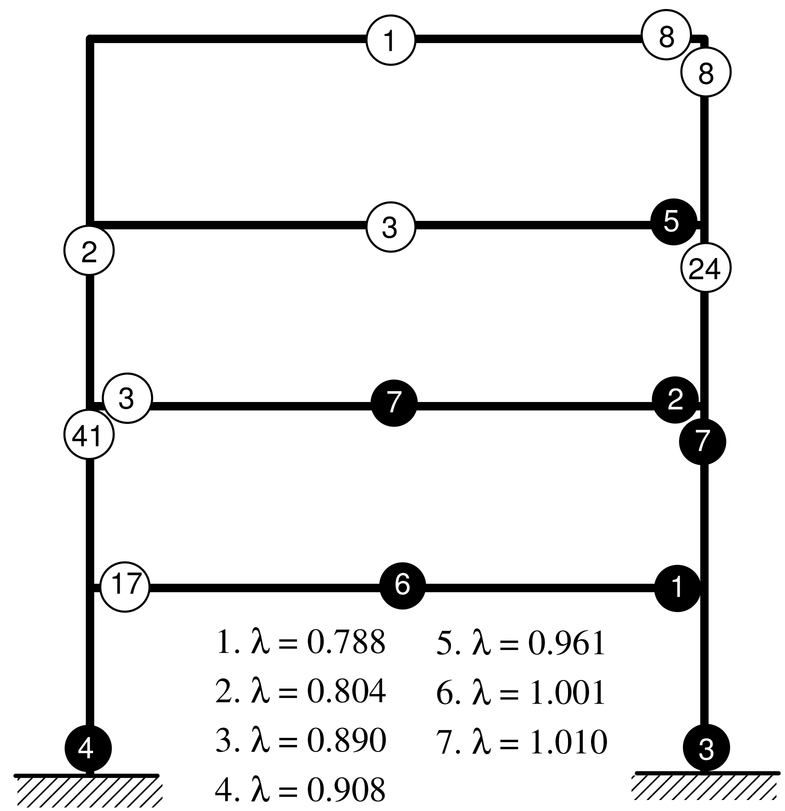
Liew Formulation (λ_{cr} = 1.055)



1. $\lambda = 0.903$
2. $\lambda = 0.911$
3. $\lambda = 1.037$
4. $\lambda = 1.046$
5. $\lambda = 1.052$
6. $\lambda = 1.055$

CS-ASA

Chan-Chui Formulation (λ_{cr} = 1.010)



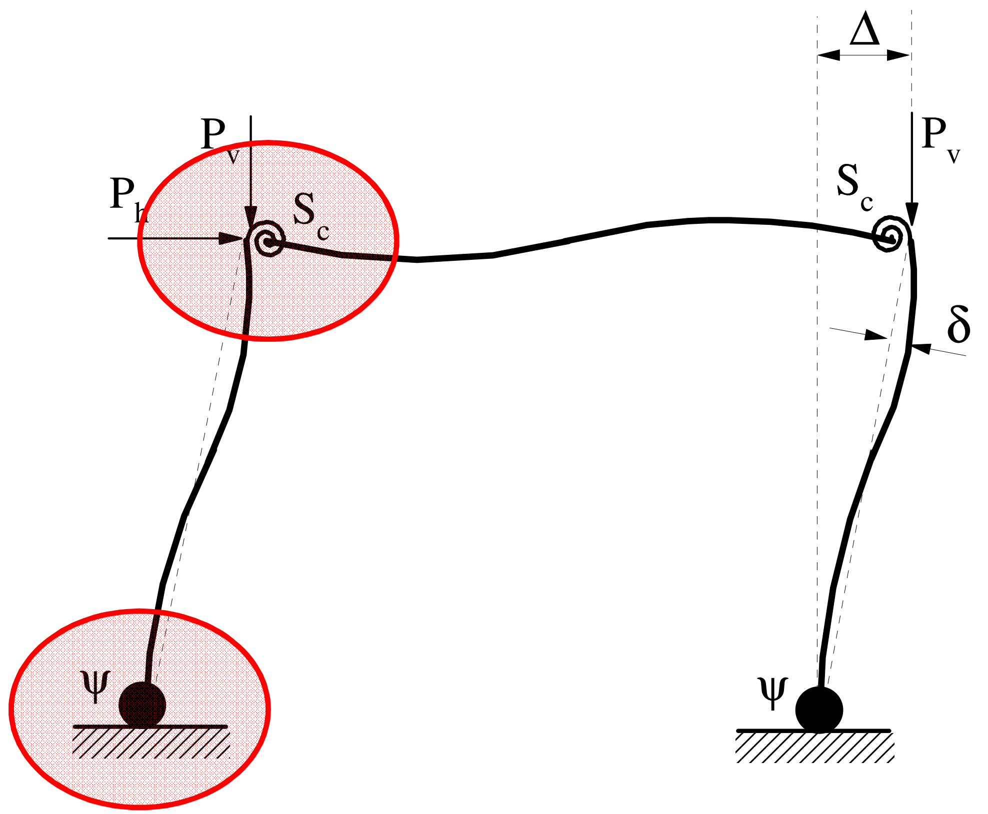
1. $\lambda = 0.788$
2. $\lambda = 0.804$
3. $\lambda = 0.890$
4. $\lambda = 0.908$
5. $\lambda = 0.961$
6. $\lambda = 1.001$
7. $\lambda = 1.010$

$\alpha = 0.5$

n Sequence of plastic-hinge formation

p % bending plasticity

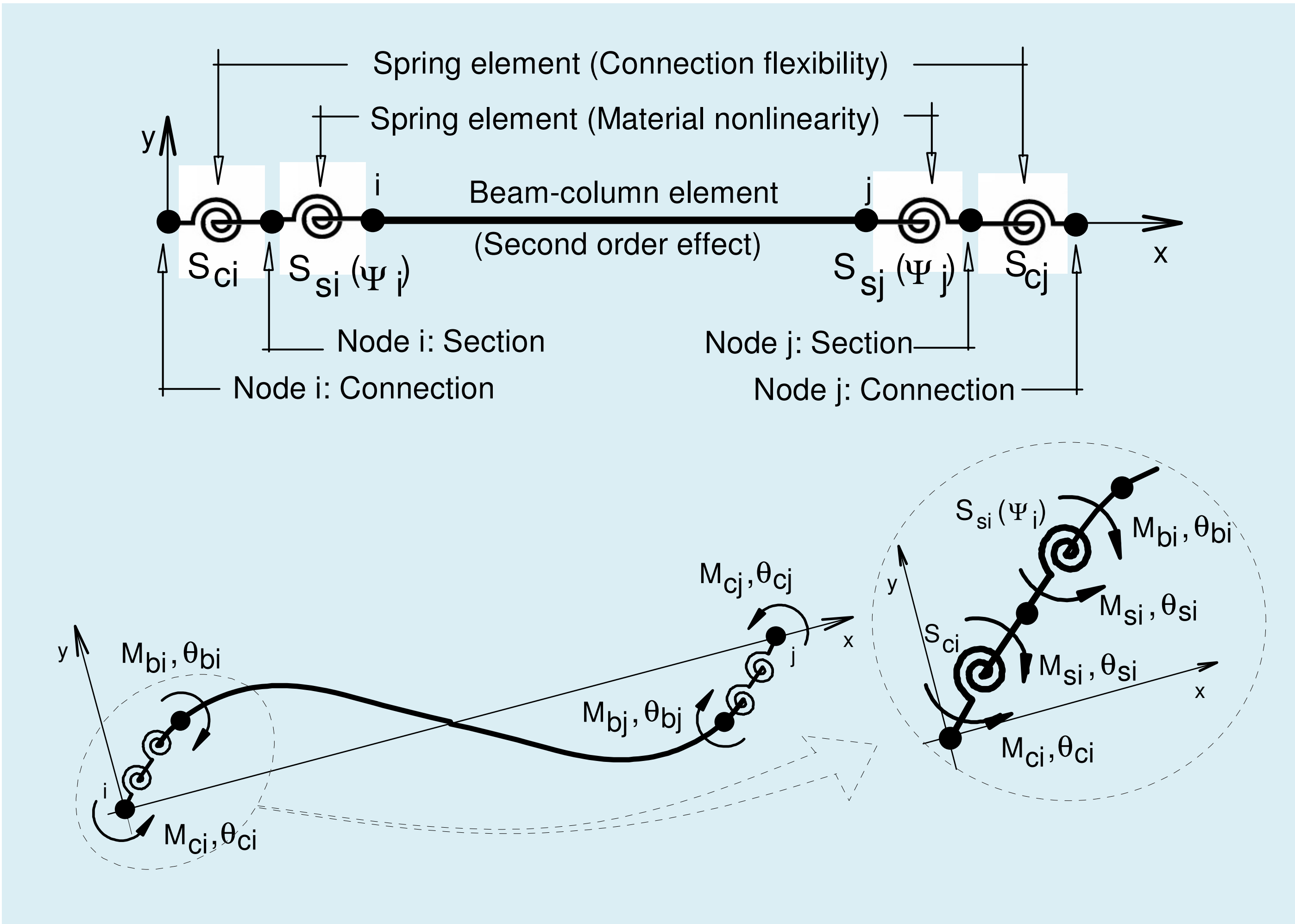
Advanced Analysis



Basic Considerations...

- The hybrid FE
- Equilibrium conditions
- Relation force-displacement
- Numerical Application

The Hybrid FE



Equilibrium Conditions

Beam-column element

$$\begin{Bmatrix} \Delta M_{bi} \\ \Delta M_{bj} \end{Bmatrix} = \begin{bmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{bmatrix} \begin{Bmatrix} \Delta \theta_{bi} \\ \Delta \theta_{bj} \end{Bmatrix}$$

Geometric nonlinear formulation

Spring element (connection)

$$\begin{Bmatrix} \Delta M_{ci} \\ \Delta M_{si} \end{Bmatrix} = \begin{bmatrix} S_{ci} & -S_{ci} \\ -S_{ci} & S_{ci} \end{bmatrix} \begin{Bmatrix} \Delta \theta_{ci} \\ \Delta \theta_{si} \end{Bmatrix}$$

S_c : Connection stiffness

Spring element (section)

$$\begin{Bmatrix} \Delta M_{si} \\ \Delta M_{bi} \end{Bmatrix} = \begin{bmatrix} S_{si} & -S_{si} \\ -S_{si} & S_{si} \end{bmatrix} \begin{Bmatrix} \Delta \theta_{si} \\ \Delta \theta_{bi} \end{Bmatrix}$$

S_s : Section stiffness

Relation Force-Displacement

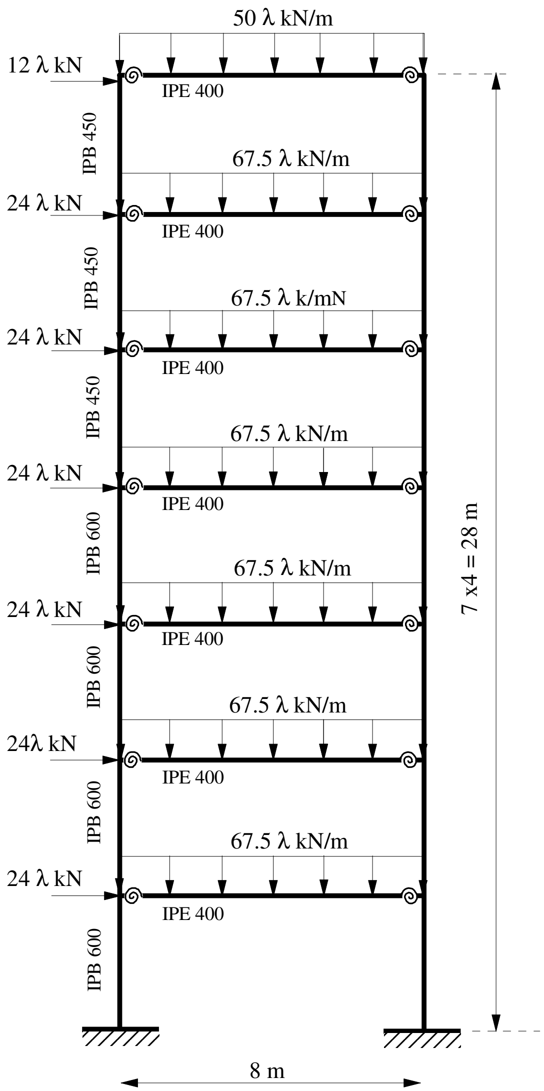
$$\begin{Bmatrix} \Delta P \\ \Delta M_i \\ \Delta M_j \end{Bmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & S_{csi} - \dfrac{S_{csi}^2}{\beta_{cs}}\left(S_{csj} + K_{jj}\right) & S_{csi} S_{csj} \dfrac{K_{ij}}{\beta_{cs}} \\ 0 & S_{csj} S_{csi} \dfrac{K_{ji}}{\beta_{cs}} & S_{csj} - \dfrac{S_{csj}^2}{\beta_{cs}}\left(S_{csi} + K_{ii}\right) \end{bmatrix} \begin{Bmatrix} \Delta\delta \\ \Delta\theta_i \\ \Delta\theta_j \end{Bmatrix}$$

$$S_{cs} = \frac{S_c S_s}{(S_c + S_s)}$$

Combined effect of connection flexibility and member plasticity

$$\beta_{cs} = (S_{csi} + K_{ii})(S_{csj} + K_{jj}) - K_{ij} K_{ji}$$

Numerical Example



Seven-Storey Frame

Sekulovik and Nefoska (2008)

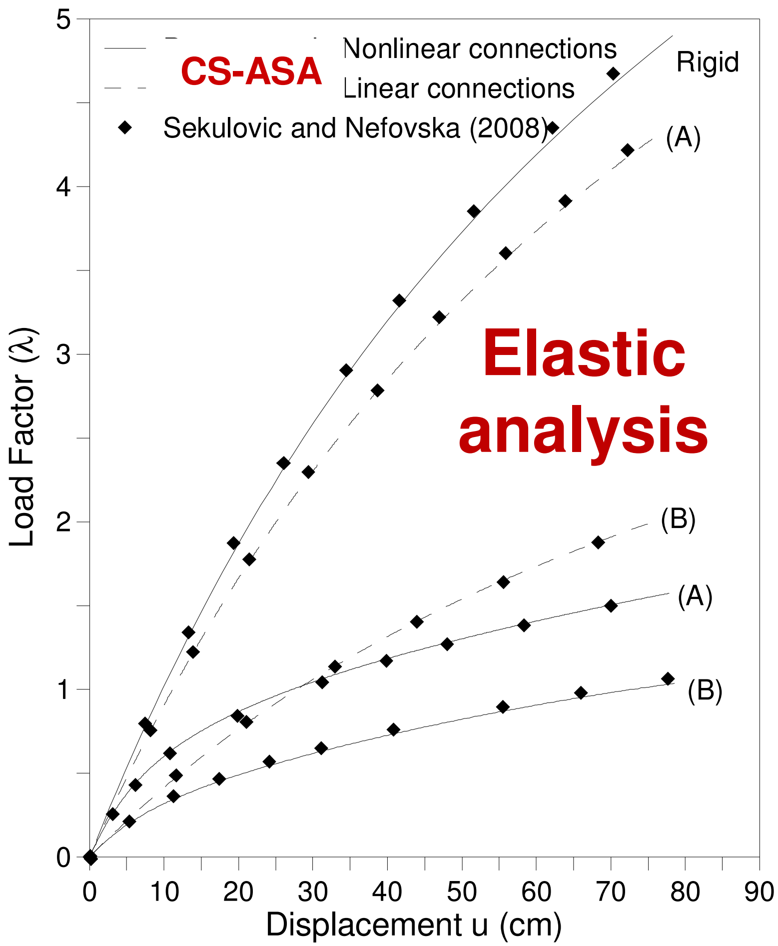
Connection	k (kNm/rad)	M_u(kNm)	n	k_p
A	200000	450.0	0.65	0.0
B	12430	101.7	1.50	0.0

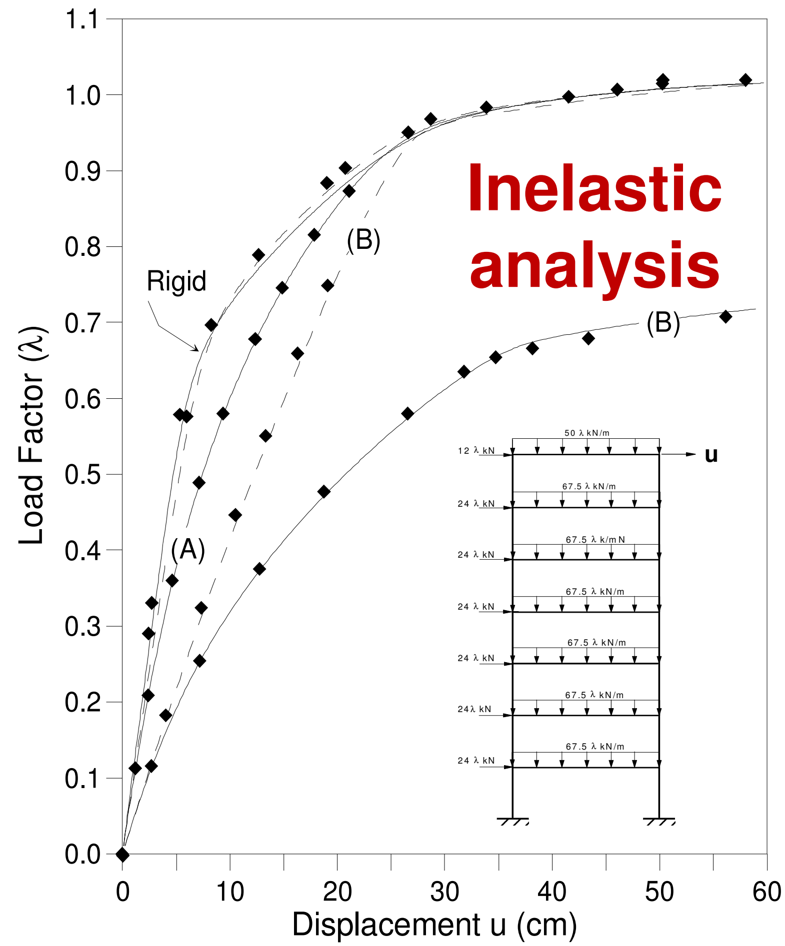
Linear joint model (S_c constant)

Nonlinear joint model : Richard-Abbott

Beams: 4 finite elements ($\sigma_r = 0.3\ \sigma_y$)
Columns: 2 finite elements ($\sigma_r = 0.5\ \sigma_y$)
$E = 205\ kN/mm^2$
$\sigma_y = 235\ N/mm^2$

Load-Displacement Curves





Nonlinear Solvers

Basic Considerations…

- **Incremental-iterative technique**
- **Important steps**
- **General algorithm**

Incremental-iterative Technique

Load increments: $j = 1,2,....,n_{máx}$

Predictor: $\begin{cases} \Delta\lambda^0 \Rightarrow \text{Load Increment Strategy} \\ \Delta\mathbf{U}^0 = \mathbf{K}_\mathbf{T}^{-1}\,\Delta\lambda^0\mathbf{F}_\mathbf{r} \end{cases}$

Equilibrium iterations: $k = 1,2,....,i_{máx}$

Corrector: $\begin{cases} \delta\lambda^k \Rightarrow \text{Iteration Strategy} \\ \delta\mathbf{U}^k = \delta\mathbf{U}_\mathbf{g}^k + \delta\lambda^k\,\delta\mathbf{U}_\mathrm{r}^k \end{cases}$

Stop iteration when: $\|\mathbf{g}\| \leq \varepsilon\,\Delta\lambda\,\|\mathbf{F}_\mathbf{r}\|$

Stop load increment when: $j = n_{máx}$

Important Steps

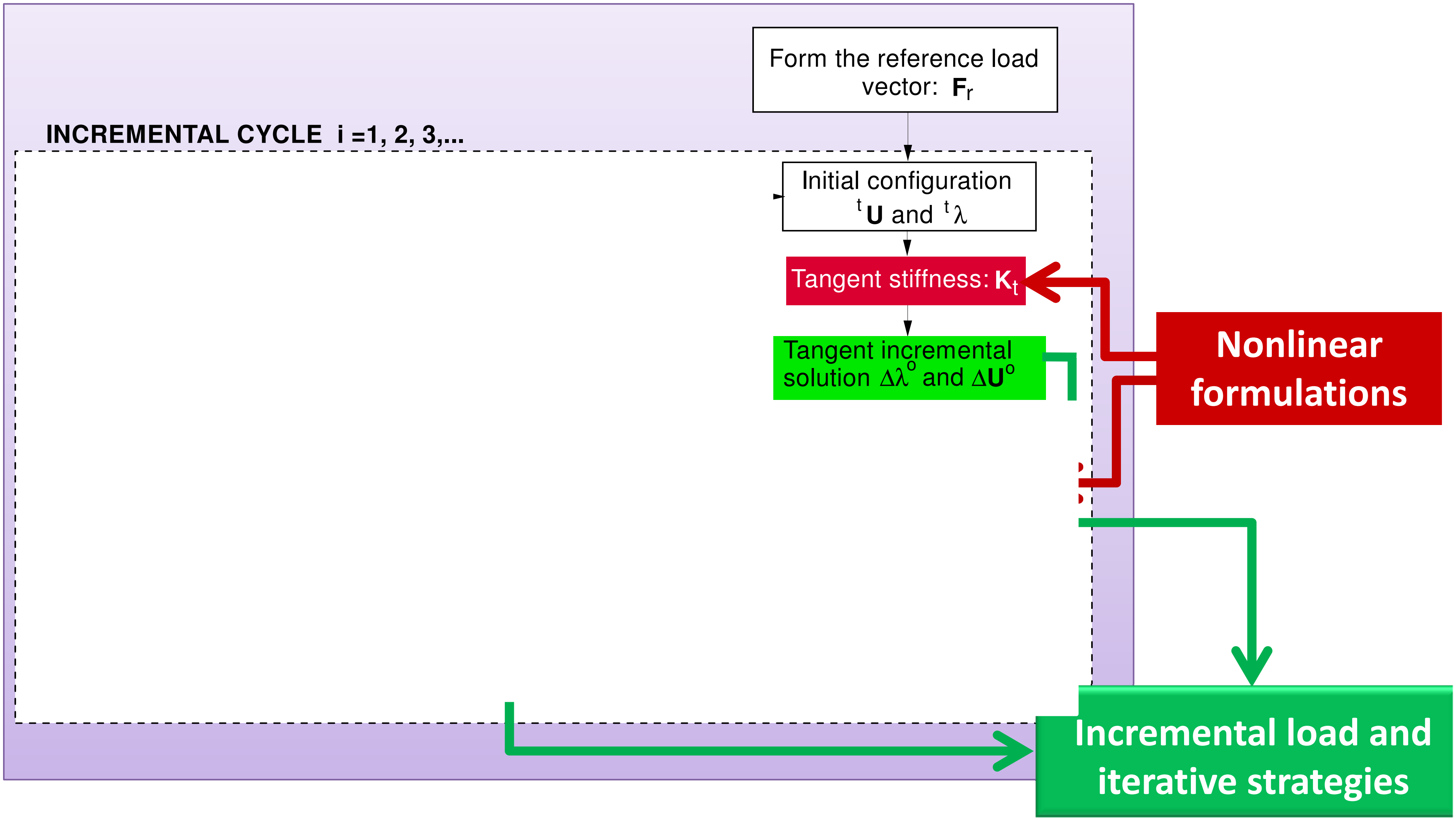
Step 1: - Tangent incremental solution: $\Delta\lambda^0$ and $\Delta\mathbf{U}^0$

- Selection of a suitable external load increment
- A particular strategy is called
LOAD INCREMENT STRATEGY
- **CS-ASA: 6 LIS**

Step 2: - Iterative solution: $\delta\lambda$ and $\delta\mathbf{U}$

- Additional constraint equation is required
- This constraint equation distinguishes the various iterative strategies
- A particular strategy is called
ITERATIVE STRATEGY
- **CS-ASA: 8 IS**

General Algorithm



Nonlinear Dynamic Problem

Basic Considerations...

- **Nonlinear transient equation**
- **Connections: behavior under cyclic loading**
- **Material: behavior under cyclic loading**
- **Numerical Application**

Nonlinear Transient Equation

$$\mathbf{M}\ddot{\mathbf{U}}_{(t+\Delta t)} + \mathbf{C}\dot{\mathbf{U}}_{(t+\Delta t)} + \mathbf{F}_{i(t+\Delta t)} = {}^{(t+\Delta t)}\lambda(t)\,\mathbf{F}_r$$

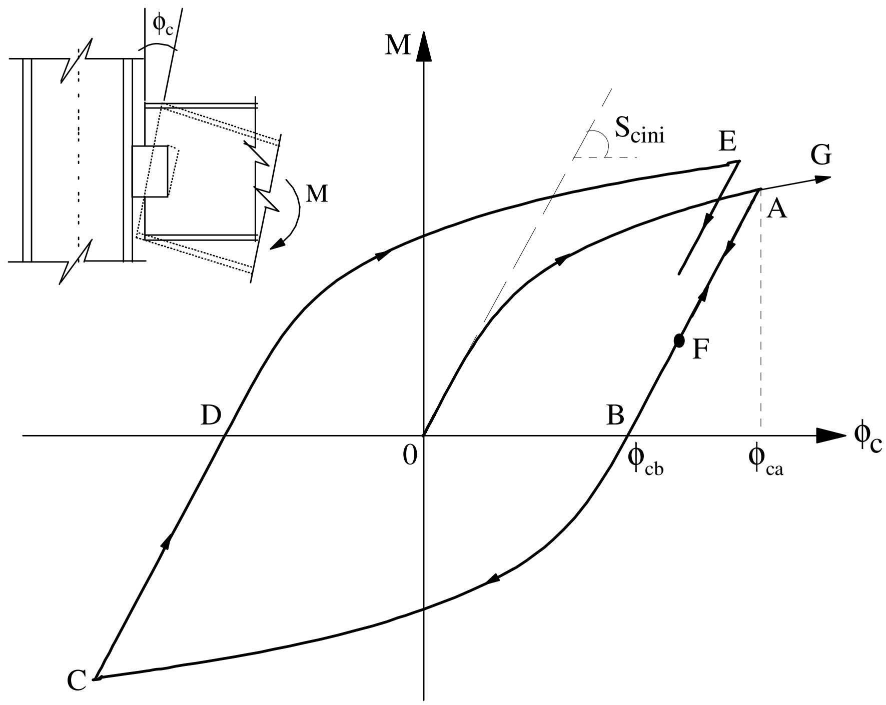
$\mathbf{F}_i$ internal forces (nodal displacements, internal forces in each element, stiffness sections, semi-rigid connections)

Nonlinear formulation

Incremental-iterative strategy

- Newmark algorithm: time integration
- Newton-Raphson method

Connections: Behavior under Cyclic Loading



Cyclic load

↓

Hysteretic Damping

Initial loading process: OA

Unloading: AB and CD

Reverse loading: BC

Reloading: DE

ϕ_{cb}: residual deformation

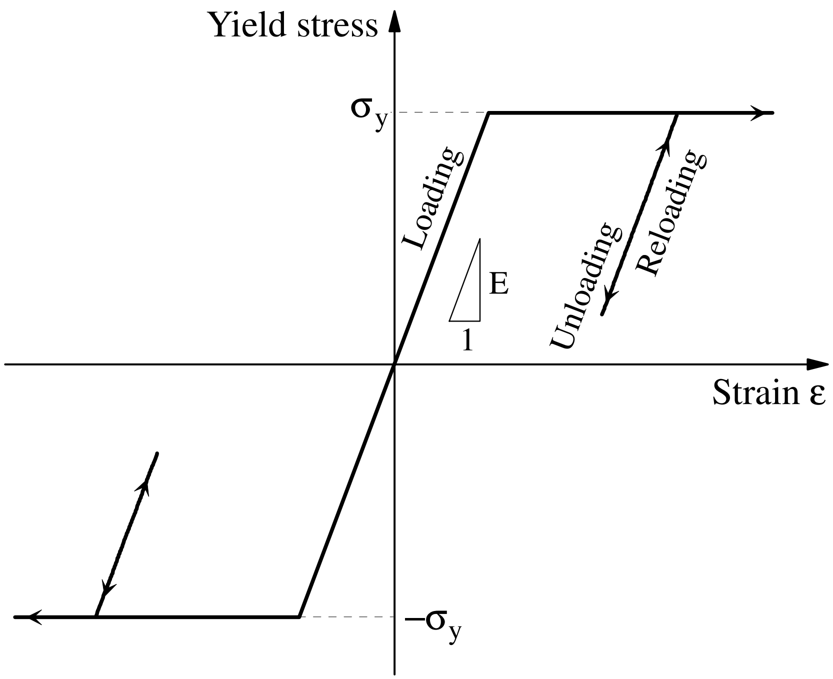
Independent Hardening Model

Loading process/Unloading process

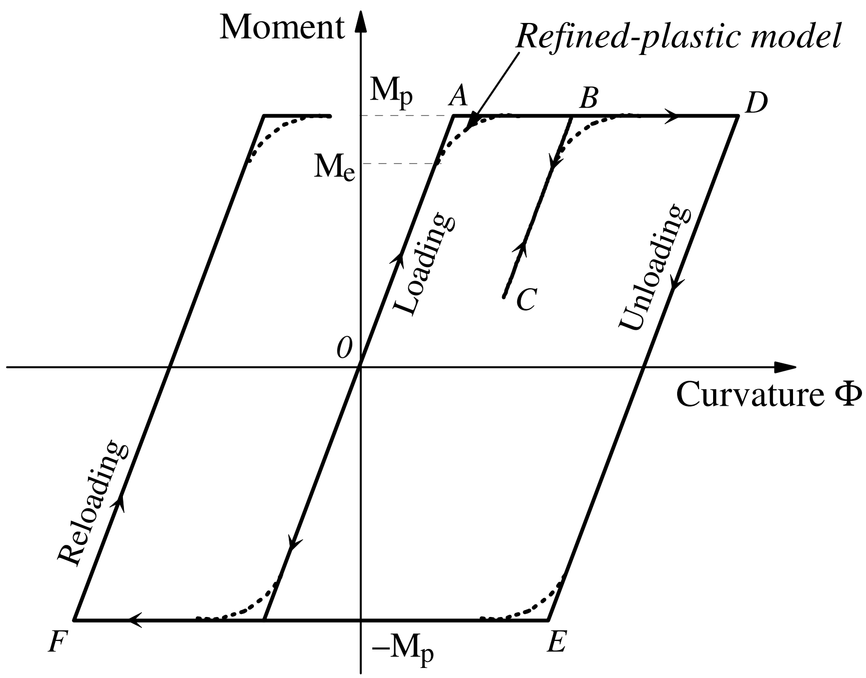
Material: Behavior under Cyclic Loading

Cyclic plasticity model

Tension-deformation relation

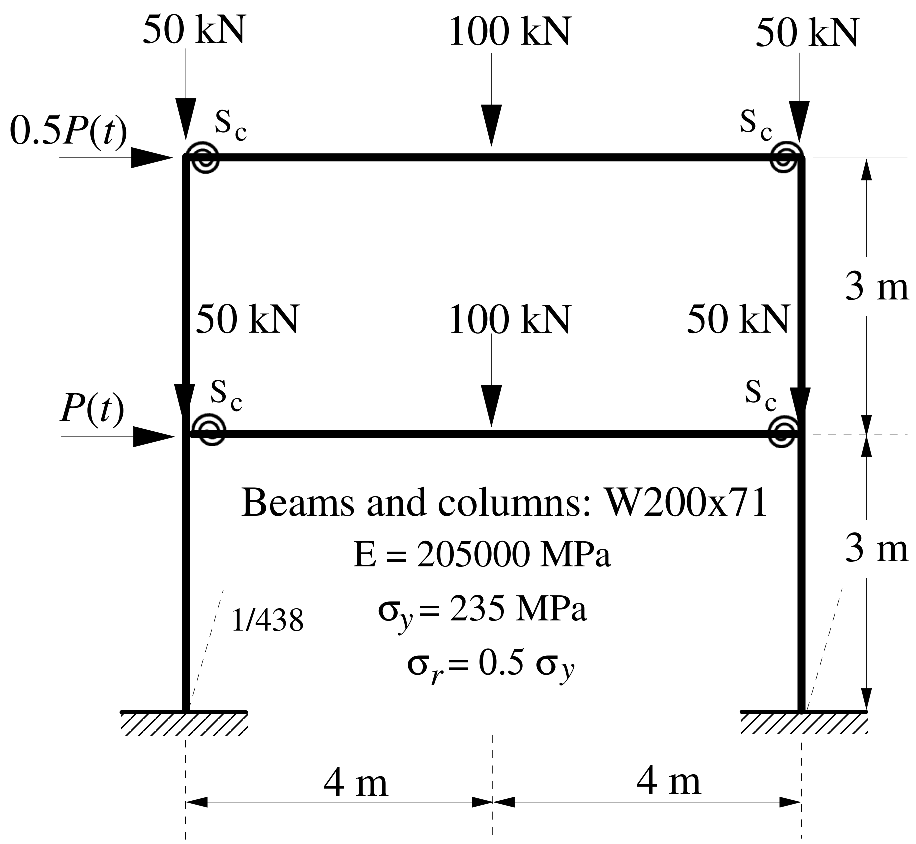


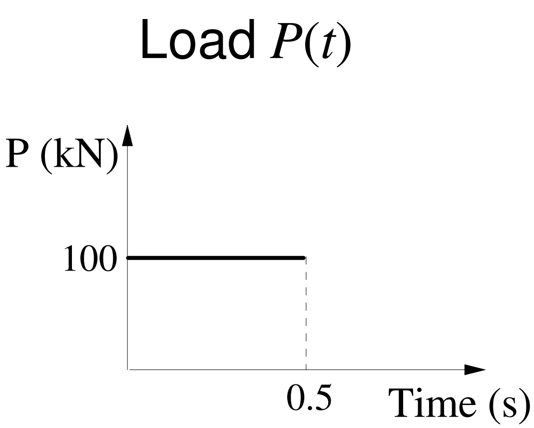
Moment-curvature relation



Strain-hardening and the Bauschinger effect are ignored

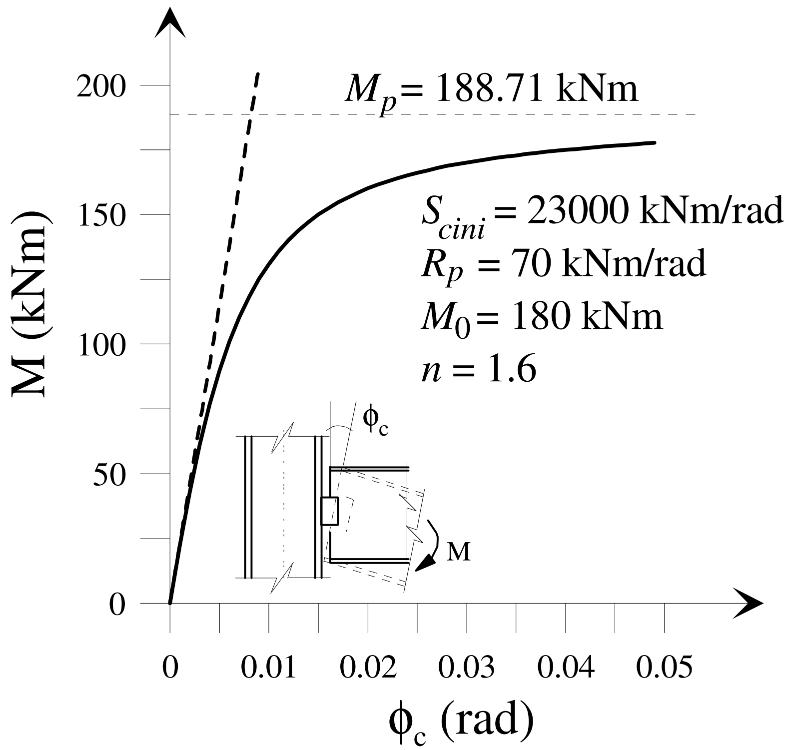
Numerical Application

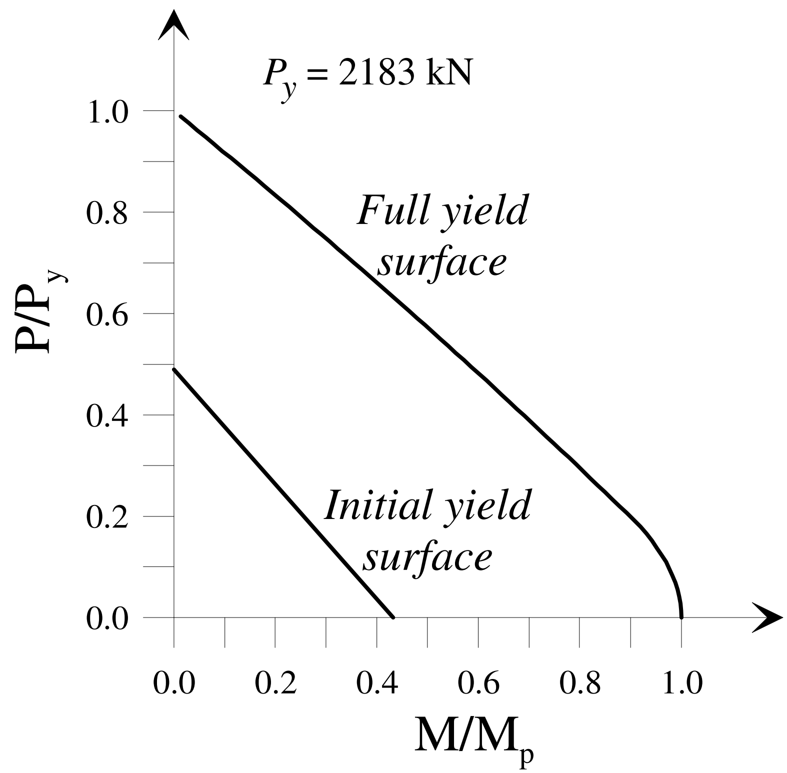




Chan and Chui (2000)

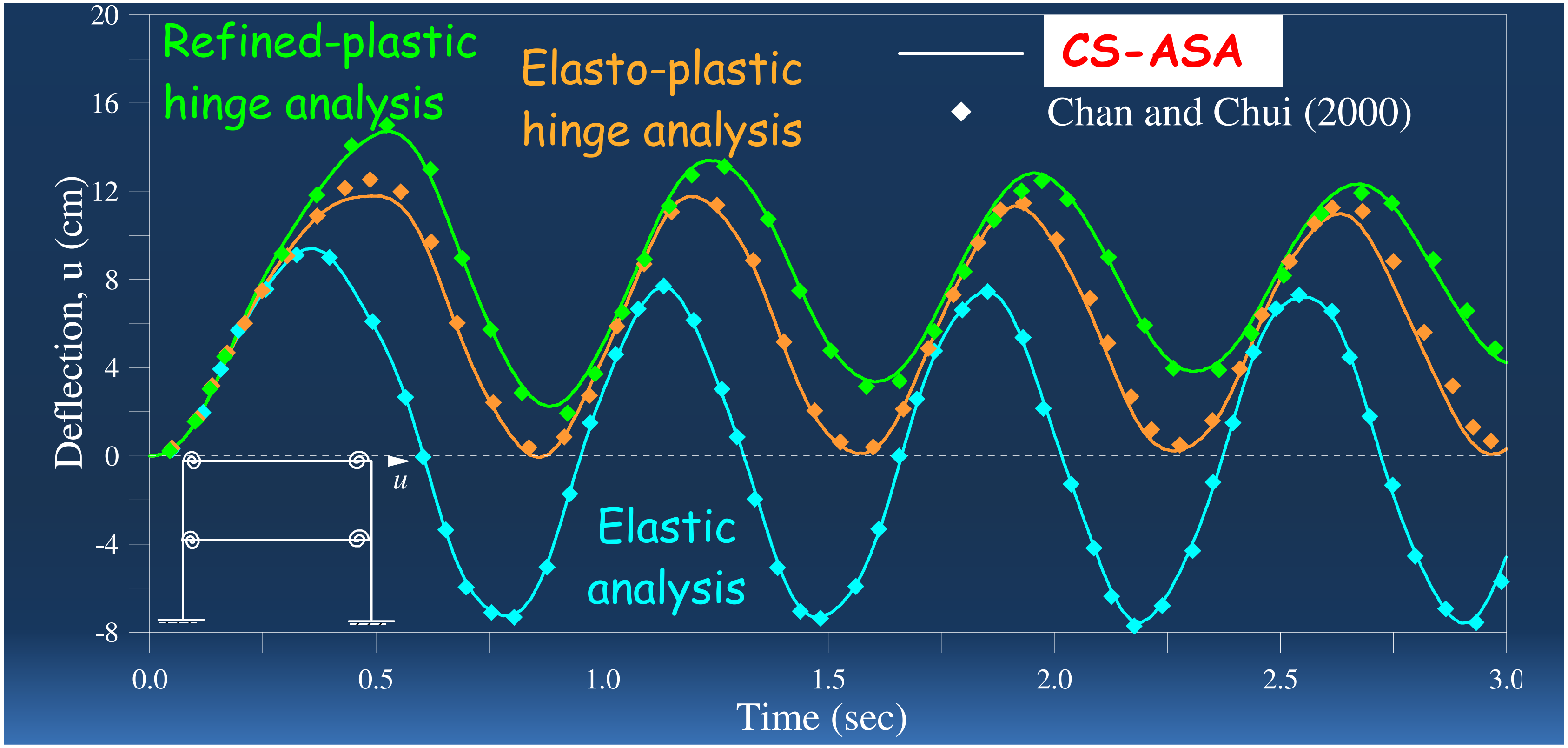
Connection behavior and yield surface





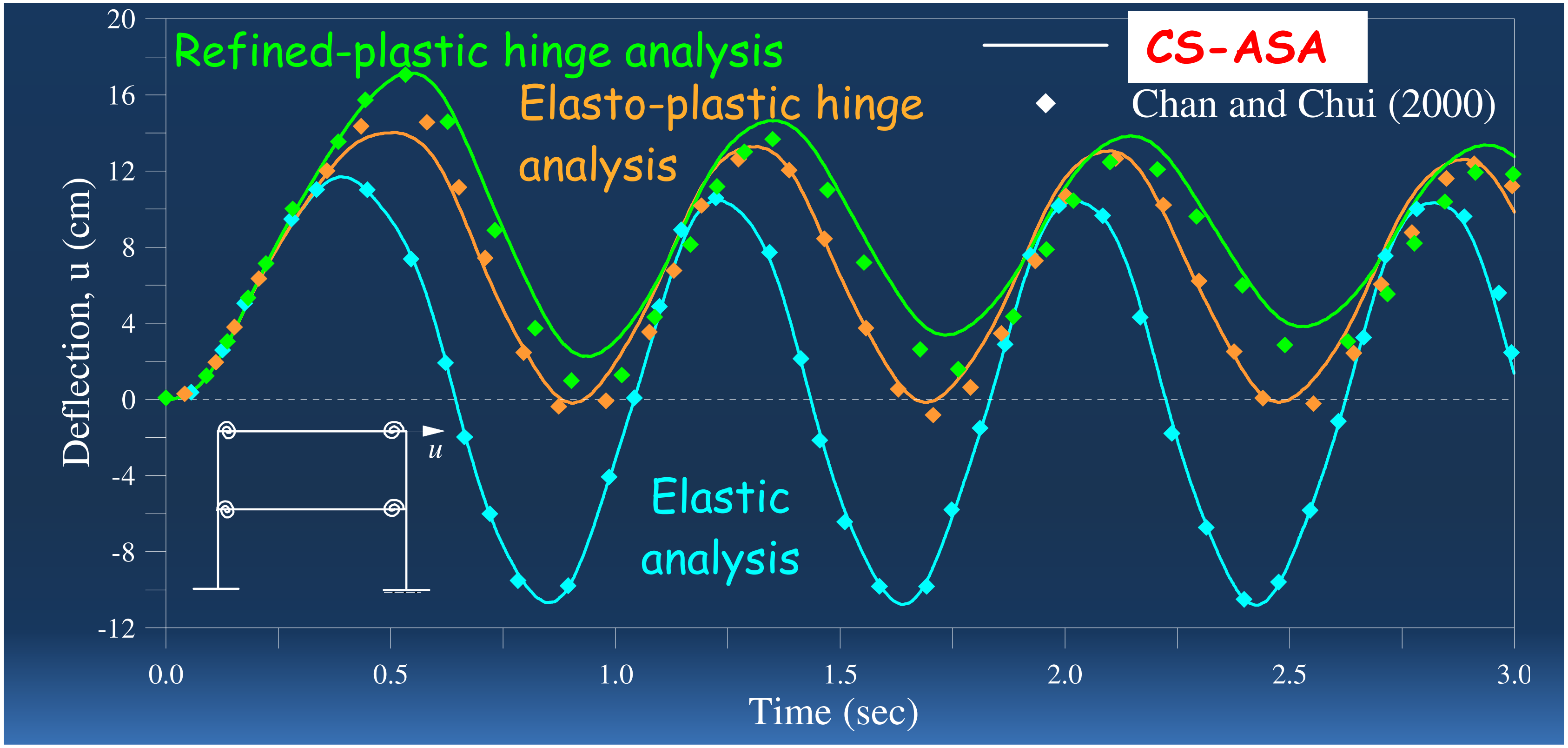
Rigid Connection

Displacement transient response



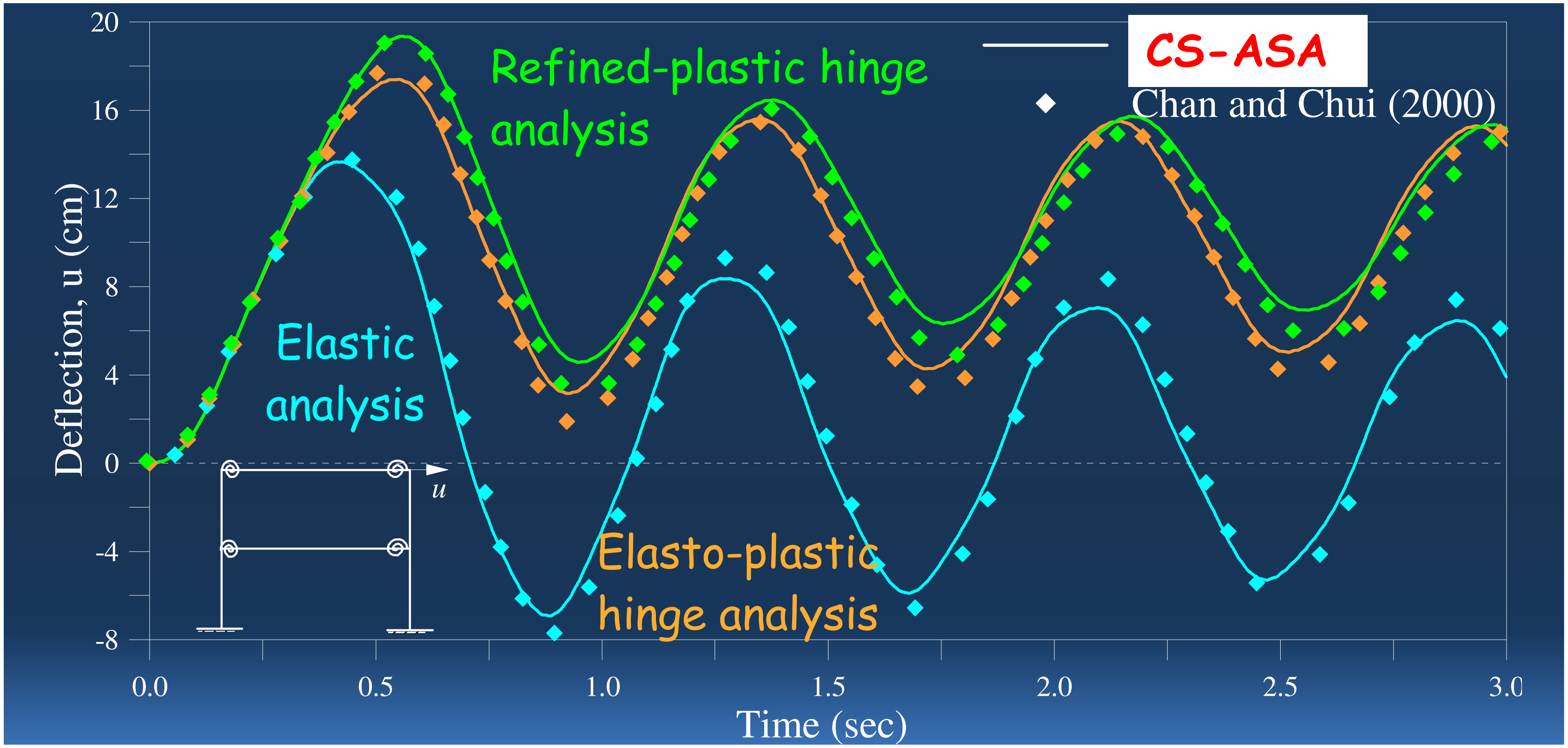
Semi-rigid Linear Connection

Displacement transient response



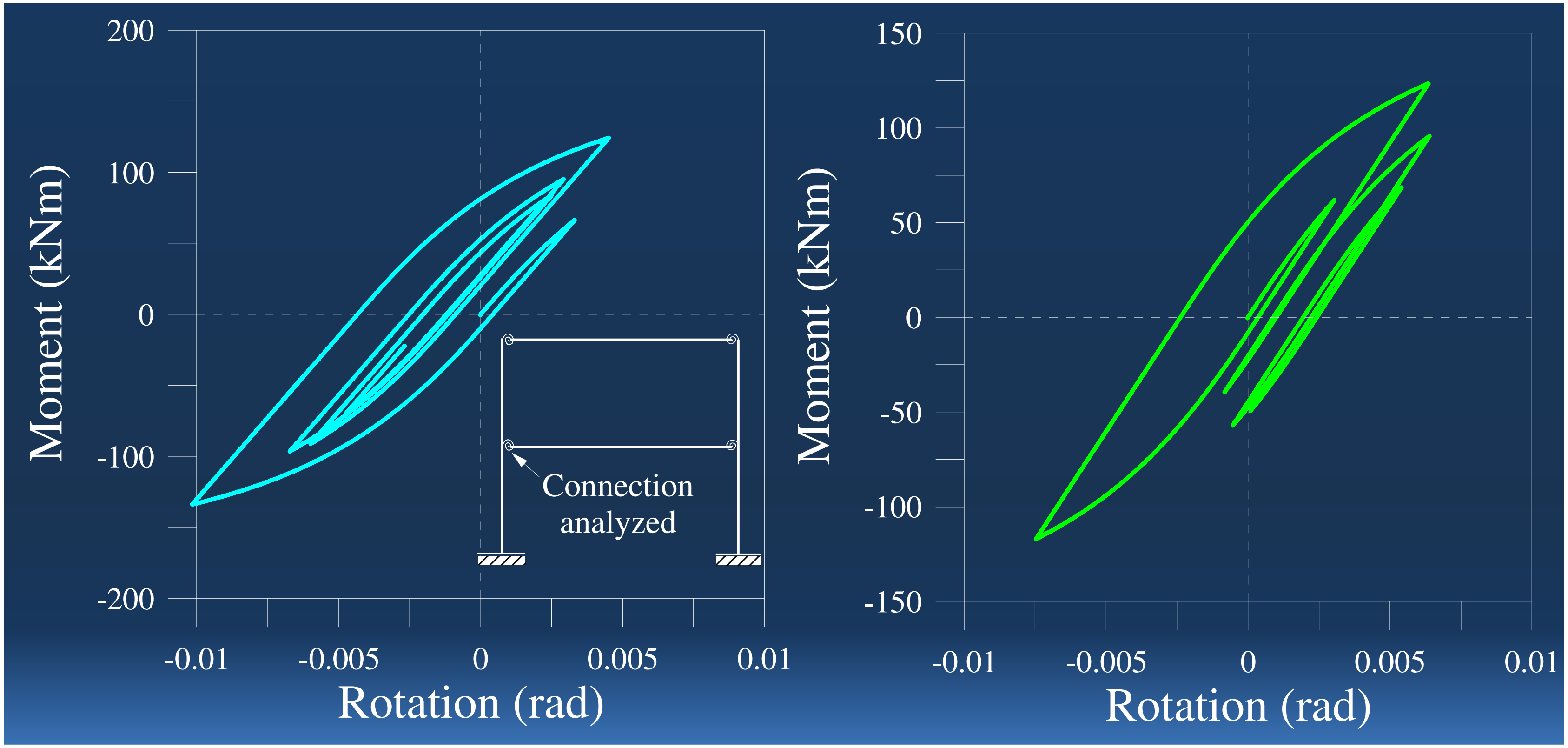
Semi-rigid Nonlinear Connection

Displacement transient response



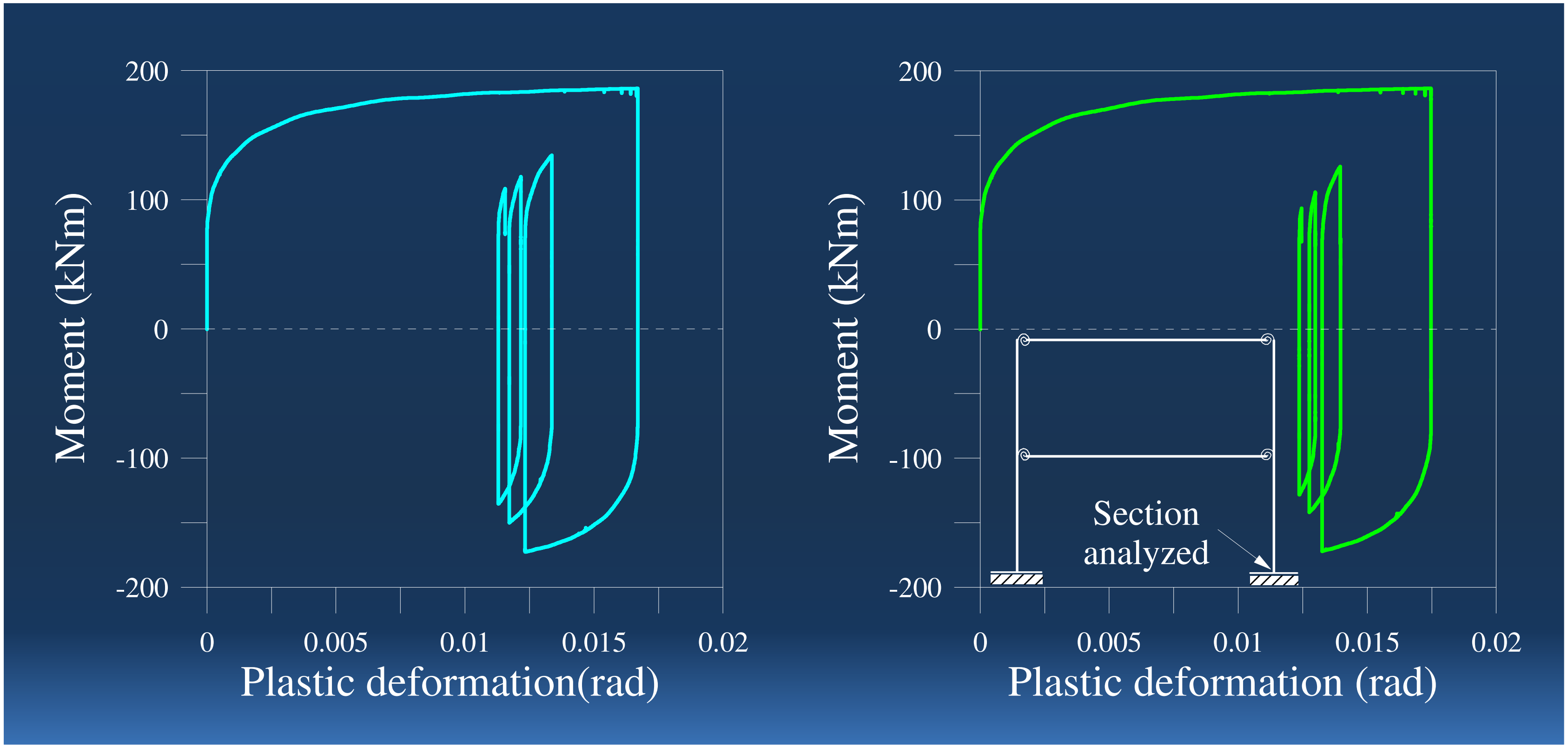
Connection Hysteretic Behavior

Elasto-plastic and Refined plastic analysis



Section Hysteretic Behavior

Linear and Nonlinear connections



Final Remarks

- Graphics pre and post–processors (Prado, 2012)
- Nonlinear dynamic problems (refined plastic-hinge approach)
  - Energy dissipation through the plastic hinges
  - Hysteretic behavior of connection: a natural damping
- Progressive collapse of structures (procedures…)
- Contact problems (soil-structure interaction; unilateral contact)
- Mixed structures
- 3D formulations

Acknowledgments

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Advanced Analysis of Steel Frames

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