UNIVERSIDADE FEDERAL DO CEARÁ

UFOP, Departamento de Engenharia Civil
8th May 2014

PROPEC
PROGRAMA DE PÓS GRADUAÇÃO EM ENGENHARIA CIVIL

Nonlinear Multi-Scale Finite Element Modeling of the Progressive Collapse of Reinforced Concrete Structures

Lecture 7: Multilayered beam

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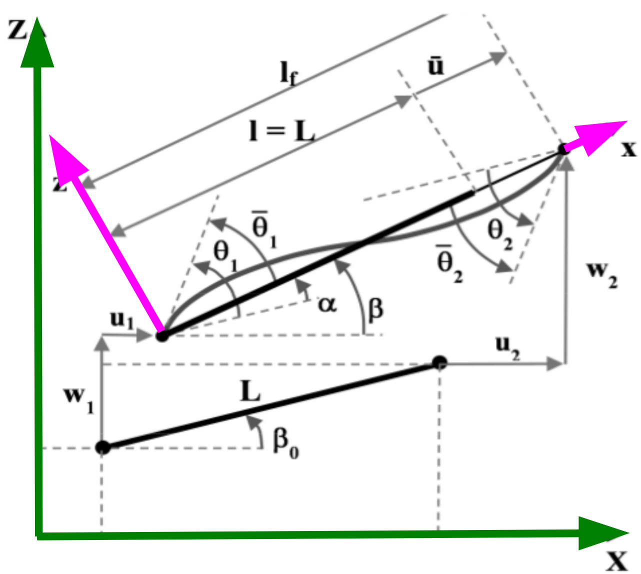
SCIENTIA VINCERE TENEBRAS
ULB

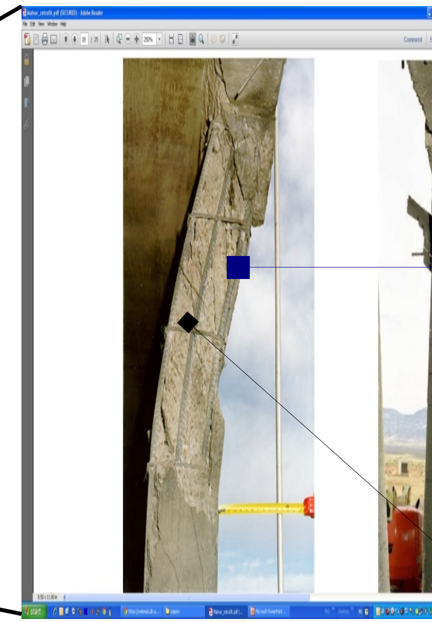
From material to structural behavior

Structure

[Experimental blast test, Crawford et al. 2001]

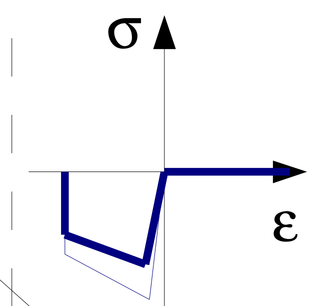


Material microstructure

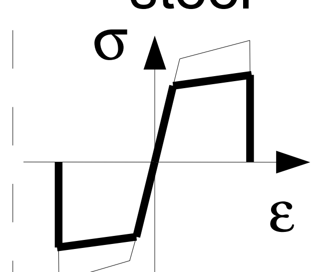


Material behavior

concrete



steel



?

Constitutive relationship for RC?

$$\mathbf{q}^T_{global} = \{u_1 \quad w_1 \quad \theta_1 \quad u_2 \quad w_2 \quad \theta_2\}$$

naturally decoupled from rigid rotation

$$\mathbf{q}_{local}{}^T = \{\bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2\}$$

conjugated

conjugated

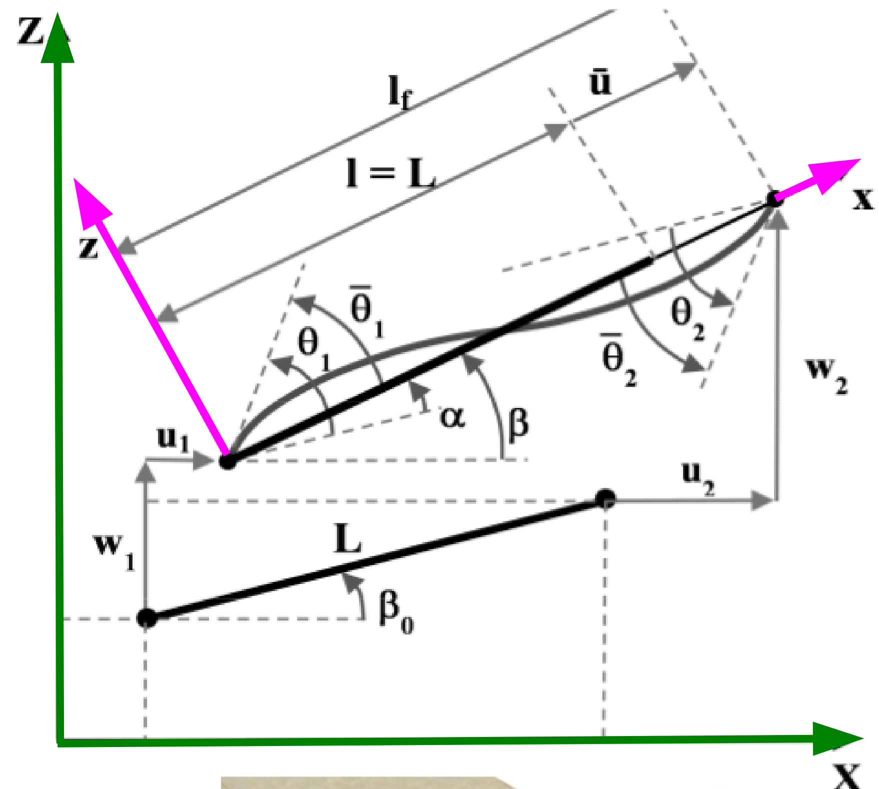
$$\left.\begin{array}{c} u_{local} \\ w_{local} \end{array}\right\}$$

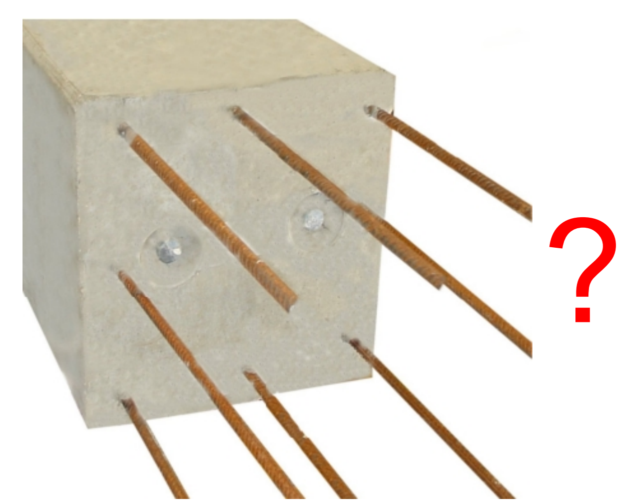
Constitutive law?

$$\begin{bmatrix} \bar{\varepsilon} \\ \chi \end{bmatrix} \rightarrow \begin{bmatrix} N \\ M \end{bmatrix}$$

Gauss pt level

$$\{f_1^x \quad f_1^y \quad c_1 \quad f_2^x \quad f_2^y \quad c_2\}^T = \mathbf{T}^T \{\bar{N} \quad \bar{M}_1 \quad \bar{M}_2\}^T$$





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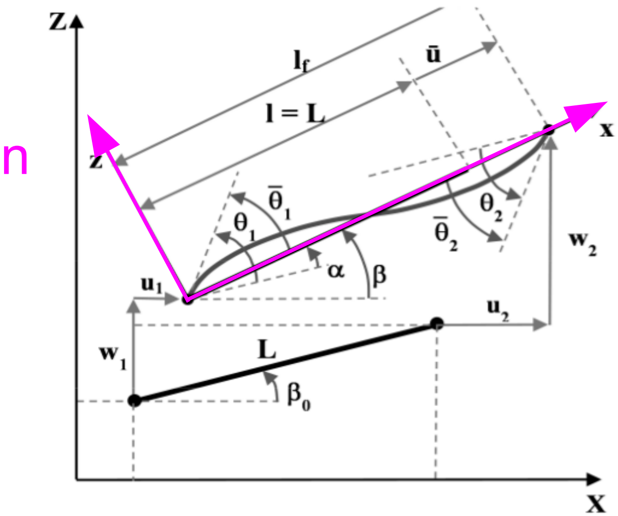
Constitutive relationship for composite section?

$$\boxed{\mathbf{q}_{local}{}^{T} = \left\{ \bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2 \right\}}$$

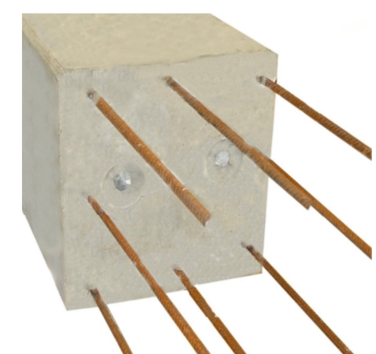
naturally decoupled from rigid rotation



$$u_{local} = \frac{x}{l}\bar{u}$$

$$w_{local} = x\left(1 - \frac{x}{l}\right)^2 \bar{\theta}_1 + \frac{x^2}{l}\left(\frac{x}{l} - 1\right)\bar{\theta}_2$$

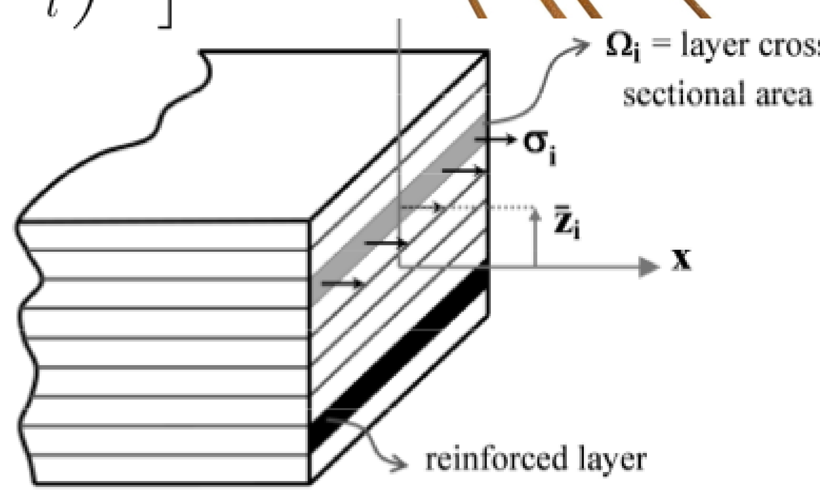
$$\begin{bmatrix} \bar{\varepsilon} \\ \chi \end{bmatrix} = \begin{bmatrix} \frac{\partial u_{local}}{\partial x} \\ \frac{\partial^2 w_{local}}{\partial x^2} \end{bmatrix}$$



Axial strain at a given beam depth (Bernoulli)

$$\varepsilon(x, z) = \frac{\partial u_{local}}{\partial x} - \chi z = \frac{\bar{u}}{l} - z\left[\left(6\frac{x}{l^2} - \frac{4}{l}\right)\bar{\theta}_1 + \left(6\frac{x}{l^2} - \frac{2}{l}\right)\bar{\theta}_2\right]$$

Discretize the composite beam cross-section into layers of finite thickness!



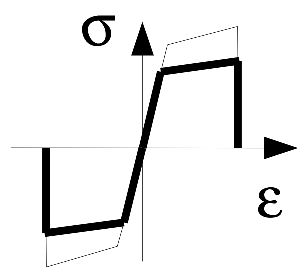
Constitutive relationship for composite section?

Axial strain at a given beam depth (Bernoulli)

$$\varepsilon(x,z) = \frac{\partial u_{local}}{\partial x} - \chi z = \frac{\bar{u}}{l} - z\left[\left(6\frac{x}{l^2} - \frac{4}{l}\right)\bar{\theta}_1 + \left(6\frac{x}{l^2} - \frac{2}{l}\right)\bar{\theta}_2\right]$$

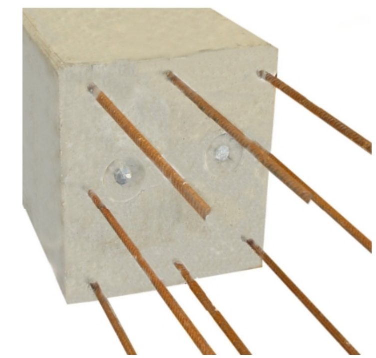
Hyp #1: Perfect adherence between concrete and steel

$$\epsilon(x,z)_{steel} = \epsilon(x,z)$$

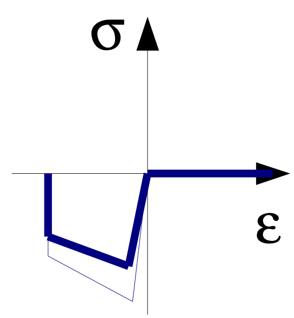


$\sigma(x,z)_{steel}$

(possible R-M)



$$\epsilon(x,z)_{concr} = \epsilon(x,z)$$

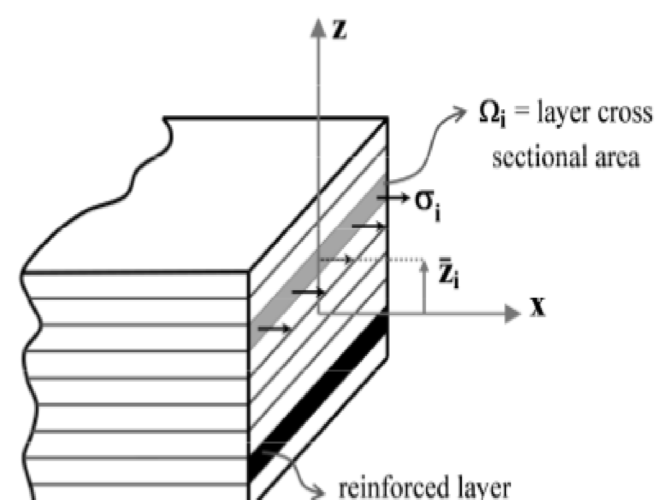


$\sigma(x,z)_{concr}$

(possible R-M)

Hyp #2: Weighted average stress of a layer

$$\sigma_{i,total} = \frac{\Omega_i - \Omega_{i,steel}}{\Omega_i}\sigma_{i,conc} + \frac{\Omega_{i,steel}}{\Omega_i}\sigma_{i,steel}$$



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Constitutive relationship for composite section?

Axial strain at a given beam depth (Bernoulli)

$$\varepsilon(x,z)=\frac{\partial u_{local}}{\partial x}-\chi z=\frac{\bar{u}}{l}-z\left[\left(6\frac{x}{l^2}-\frac{4}{l}\right)\bar{\theta}_1+\left(6\frac{x}{l^2}-\frac{2}{l}\right)\bar{\theta}_2\right]$$

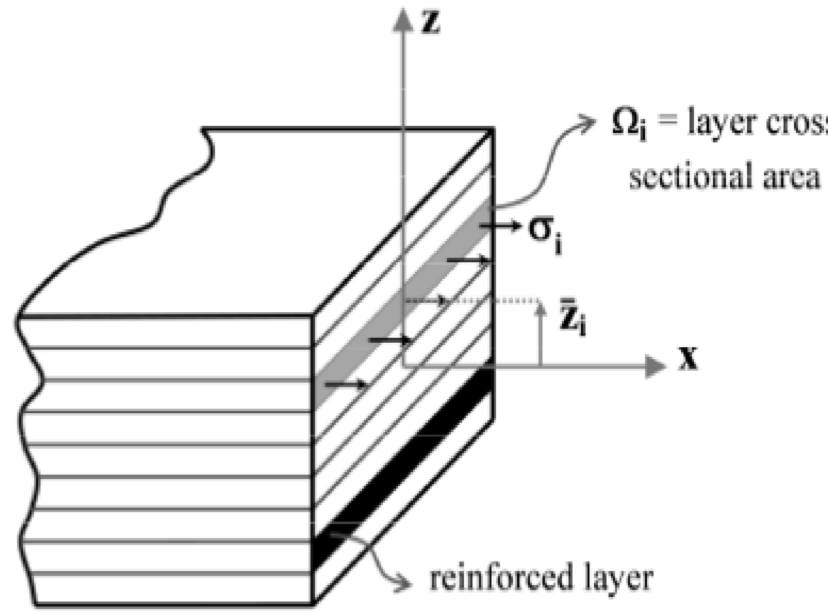
Weighted average stress of a layer

$$\sigma_{i,total}=\frac{\Omega_i-\Omega_{i,steel}}{\Omega_i}\sigma_{i,conc}+\frac{\Omega_{i,steel}}{\Omega_i}\sigma_{i,steel}$$

Generalized stresses at Gauss points

$$N=\int_{a_{cor}}\sigma\,da \longrightarrow N=\sum_i \sigma_{i,total}\Omega_i$$

$$M=-\int_{a_{cor}}\sigma z\,da \longrightarrow M=\sum_i -\bar{z}_i\sigma_{i,total}\Omega_i$$



$$\bar{N}=\int_{v_{cor}}\frac{\sigma}{l}\,dv=\int_{l}\frac{N}{l}\,dl$$

$$\bar{M}_1=-\int_{v_{cor}}\sigma z\left(6\frac{x}{l^2}-\frac{4}{l}\right)dv=\int_{l}M\left(6\frac{x}{l^2}-\frac{4}{l}\right)dl$$

$$\bar{M}_2=-\int_{v_{cor}}\sigma z\left(6\frac{x}{l^2}-\frac{2}{l}\right)dv=\int_{l}M\left(6\frac{x}{l^2}-\frac{2}{l}\right)dl$$

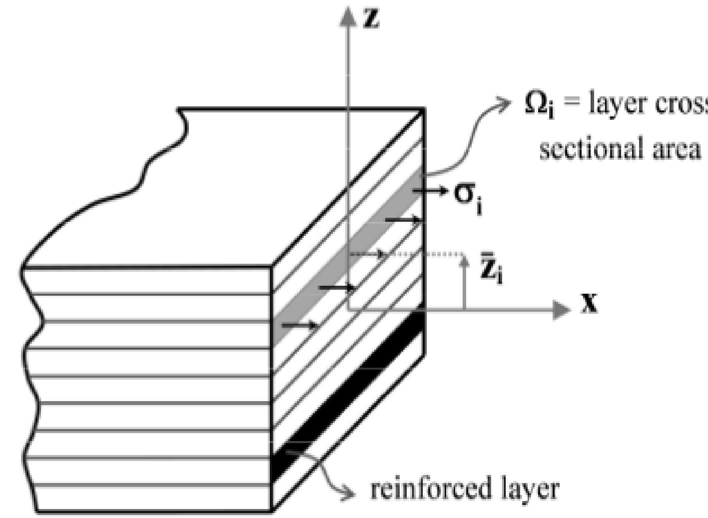
Constitutive relationship obtained computationally!

Structural stiffness matrix

$$\mathbf{K}_{global} = \mathbf{T}^T \mathbf{K}_{local} \mathbf{T} + \bar{N} \frac{\mathbf{z}\mathbf{z}^T}{l_f} + (\bar{M}_1 + \bar{M}_2) \frac{1}{l_f^2} \left(\mathbf{r}\mathbf{z}^T + \mathbf{z}\mathbf{r}^T \right)$$

Contribution of the stress variation

Other terms: contribution of the change in the geometry



$$\mathbf{K}_{local} = \int_l \mathbf{B}^T \mathbf{H} \mathbf{B} \, dl$$

sectional stiffness

$$\mathbf{H} = \frac{\partial \mathbf{\Sigma}^{gen}}{\partial \mathbf{E}^{gen}} = \begin{bmatrix} \sum_i H_i \Omega_i & -\sum_i H_i \bar{z}_i \Omega_i \\ -\sum_i H_i \bar{z}_i \Omega_i & -\sum_i H_i \bar{z}_i^2 \Omega_i \end{bmatrix}$$

with
$$\begin{cases} N = \sum_i \sigma_{i,total} \Omega_i \\ M = \sum_i - \bar{z}_i \sigma_{i,total} \Omega_i \end{cases}$$
and
$$\mathbf{E}^{gen} = \begin{bmatrix} \bar{\varepsilon} \\ \chi \end{bmatrix}$$

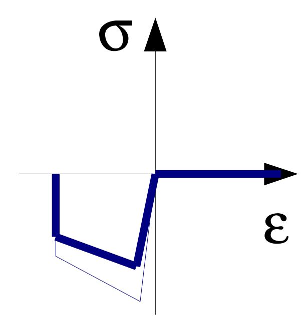
material stiffness of a layer

$$H_i = \frac{\partial \sigma_{i,total}}{\partial \varepsilon_i}$$

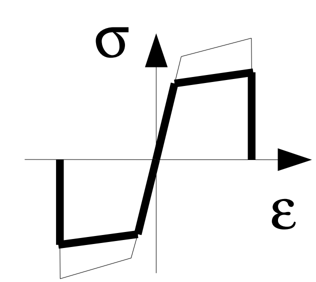
material stiffness

$$H_i = \frac{\Omega_i - \Omega_{i,steel}}{\Omega_i} L_{i,conc} + \frac{\Omega_{i,steel}}{\Omega_i} L_{i,steel}$$

since
$$\sigma_{i,total} = \frac{\Omega_i - \Omega_{i,steel}}{\Omega_i} \sigma_{i,conc} + \frac{\Omega_{i,steel}}{\Omega_i} \sigma_{i,steel}$$



$$L_{i,conc} = \frac{\partial \sigma_{i,conc}}{\partial \epsilon}$$



$$L_{i,steel} = \frac{\partial \sigma_{i,steel}}{\partial \epsilon}$$

Return Mapping at layers (1D case)

At each layer of the cross section

$$\sigma^{(k)} = \sigma_c + \mathrm{E}\,(\Delta\epsilon - \Delta\epsilon_p^{(k)})$$

$\sigma_{tr} = \sigma_c + \mathrm{E}\,\Delta\epsilon$ trial stress – elastic increment assumption

if $f_p\left(\sigma_{tr},\, {}^t\kappa\right) > 0$

$$\sigma^{(k)} = \sigma_{tr} - \mathrm{E}\,\Delta\epsilon_p^{(k)}$$

$$\frac{\sigma^{(k)} - \sigma_{tr}}{\mathrm{E}} + \Delta\epsilon_p^{(k)} = 0$$

solve using Newton-Raphson

$$\begin{cases} \dfrac{\sigma^{(k)} - \sigma_{tr}}{\mathrm{E}} + \Delta\epsilon_p^{(k)} = 0 \\ f(\sigma^{(k)},\, \kappa^{(k)}) = 0 \end{cases}$$

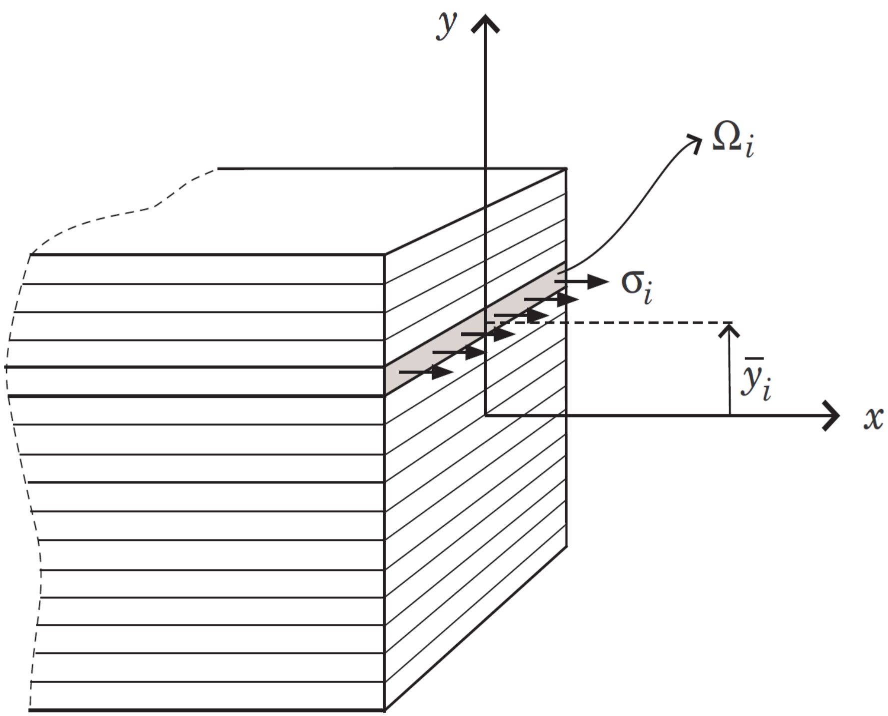
$$\begin{Bmatrix} \sigma^{(k+1)} \\ \kappa^{(k+1)} \end{Bmatrix} = \begin{Bmatrix} \sigma^{(k)} \\ \kappa^{(k)} \end{Bmatrix} - [\mathbf{J}_p(\sigma^{(k)}, \kappa^{(k)})]^{-1} \begin{Bmatrix} R^{(k)} \end{Bmatrix}$$

$$L = \frac{\partial \sigma}{\partial \epsilon}$$

with $\mathbf{J}_p(\sigma^{(k)}, \kappa^{(k)}) = \begin{bmatrix} \dfrac{\partial R_\epsilon}{\partial \sigma} & \dfrac{\partial R_\epsilon}{\partial \kappa} \\ \dfrac{\partial R_f}{\partial \sigma} & \dfrac{\partial R_f}{\partial \kappa} \end{bmatrix}$

Multilayered beam - summary

Integration point level



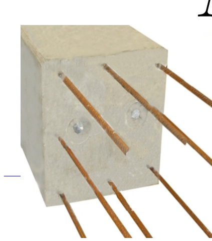
Generalized strains

Axial strain $\bar{\epsilon}$

Curvature χ

Generalized stresses

$$N = \sum \sigma_i \Omega_i$$
$$M = -\sum \sigma_i \bar{y}_i \Omega_i$$



Layer level

$$\epsilon_i = \bar{\epsilon} - \bar{y}_i \chi$$
$$\dot{\epsilon}_i = \dot{\bar{\epsilon}} - \bar{y}_i \dot{\chi}$$

Constitutive laws

$$\sigma_i = \frac{\Omega_i - \Omega_{i,steel}}{\Omega_i} \sigma_{i,conc.} + \frac{\Omega_{i,steel}}{\Omega_i} \sigma_{i,steel}$$

weighted average stress in the layer

Solution - Return Mapping (1D case)

At each Gauss point

$$\sigma^{(k)} = \sigma_c + \mathrm{E}\,(\Delta\epsilon - \Delta\epsilon_p^{(k)})$$

$\sigma_{tr} = \sigma_c + \mathrm{E}\,\Delta\epsilon$ trial stress – elastic increment assumption

if $f_p\left(\sigma_{tr},\, {}^t\kappa\right) > 0$

$$\sigma^{(k)} = \sigma_{tr} - \mathrm{E}\,\Delta\epsilon_p^{(k)}$$

$$\frac{\sigma^{(k)} - \sigma_{tr}}{\mathrm{E}} + \Delta\epsilon_p^{(k)} = 0$$

solve using Newton-Raphson

$$\begin{cases} \dfrac{\sigma^{(k)} - \sigma_{tr}}{\mathrm{E}} + \Delta\epsilon_p^{(k)} = 0 \\ f(\sigma^{(k)},\, \kappa^{(k)}) = 0 \end{cases}$$

$$\left\{ \begin{array}{c} \sigma^{(k+1)} \\ \kappa^{(k+1)} \end{array} \right\} = \left\{ \begin{array}{c} \sigma^{(k)} \\ \kappa^{(k)} \end{array} \right\} - [\mathbf{J}_p(\sigma^{(k)}, \kappa^{(k)})]^{-1} \left\{ R^{(k)} \right\}$$

$$L = \frac{\partial \sigma}{\partial \epsilon}$$

with $\mathbf{J}_p(\sigma^{(k)}, \kappa^{(k)}) = \begin{bmatrix} \dfrac{\partial R_\epsilon}{\partial \sigma} & \dfrac{\partial R_\epsilon}{\partial \kappa} \\ \dfrac{\partial R_f}{\partial \sigma} & \dfrac{\partial R_f}{\partial \kappa} \end{bmatrix}$

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Loops in the solution procedure

Loop on loading (for) **1**

Initialise residual

Loop on iterations (while residual > tolerance) **2**

Assembly of stiffness (Loop on elements) **3**

Computation reaction forces

Compute internal forces (loop on elements) **4**

Compute stresses at Gauss points (Gaus pt loop) **5**

Compute stresses at layers (loop on layers) **6**

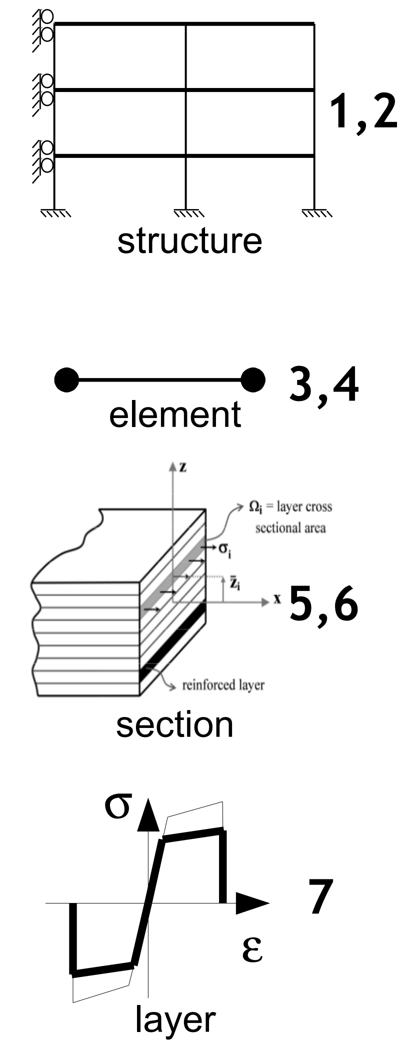
Return-Mapping at each layer (plasticity) **7**

Evaluate new residual

End of iteration loop

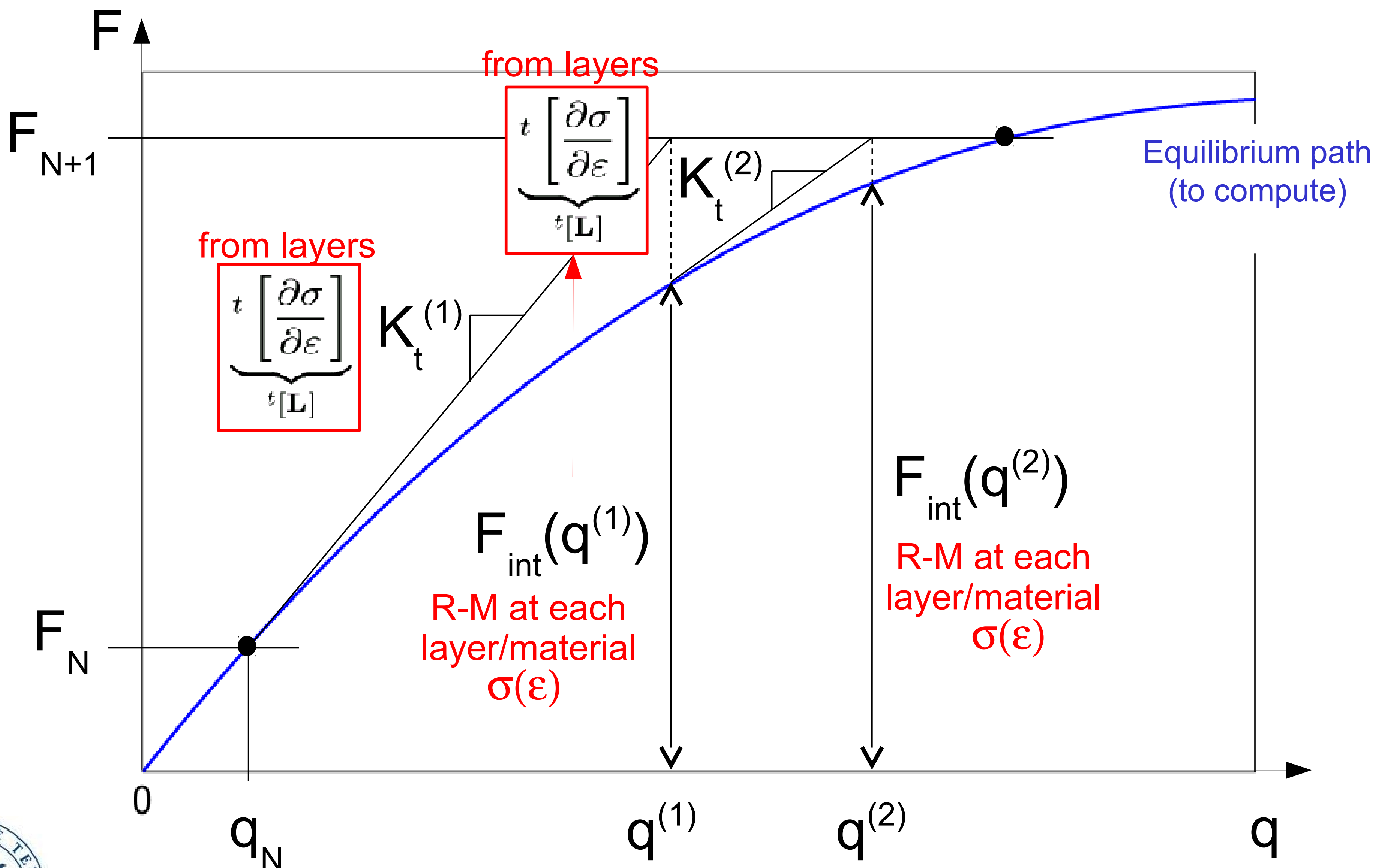
End of loop on loading

Level of the loop



Structure of a NL FE code

Graphical interpretation



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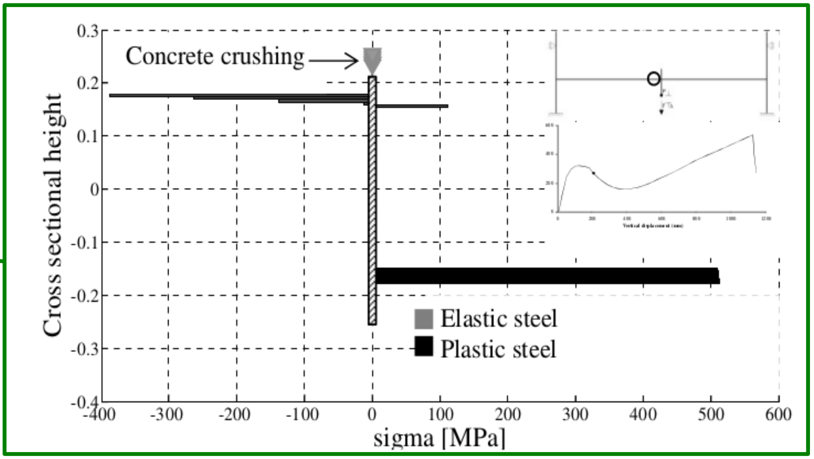
- Reduced number of assumptions
- Physically based section model
    - Reinforcement scheme
    - Reinforcement ratio
- 1D constitutive laws vs. experiments
- Extended parametric studies
- Local stresses, section states

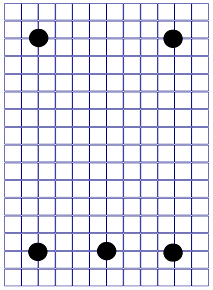
-

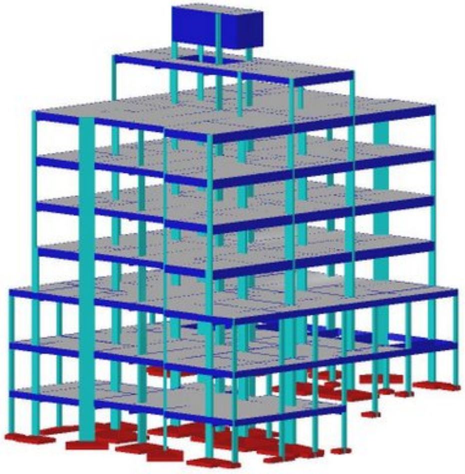
- Currently planar frames only
- High computational cost



Future extensions

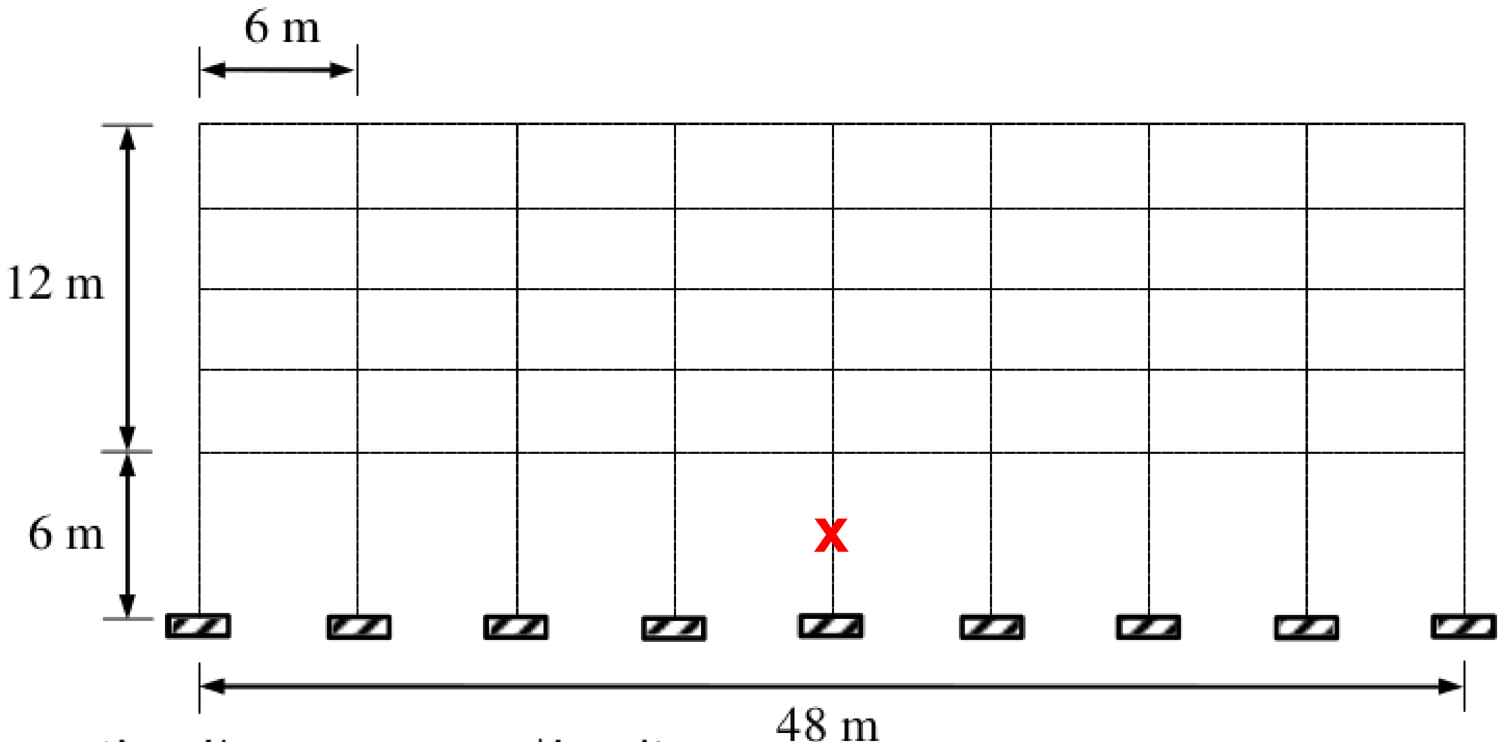
- Shear failure (Timoshenko layered beam),
- Stirrups vs. concrete strength,
- Bond slip effect,
- Damage coupled to plasticity,
- **3D extension** (material fibers),
- ….

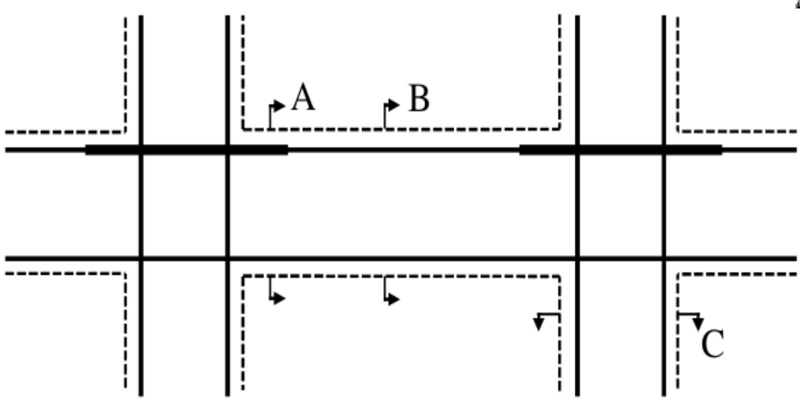




Applications – Planar Frame, Dynamics

Eurocode 2 vs NBR6118





EUROCODE based design

Loads(kN/m)	*Dead*	*Live*	*Total*
Floor Beams	43.2	18.0	52.2
Roof Beams	43.2	6.0	46.2

NBR 6118 based design

Floor Beams	18.7	12.0	24.7
Roof Beams	18.7	6.0	21.7

[6] C. Oliveira et al., submitted to IBRACON in 2014

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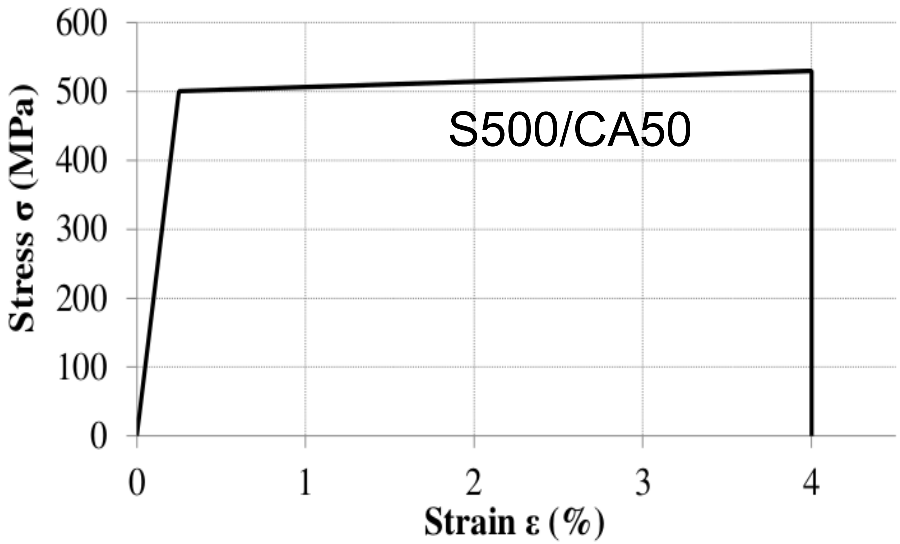
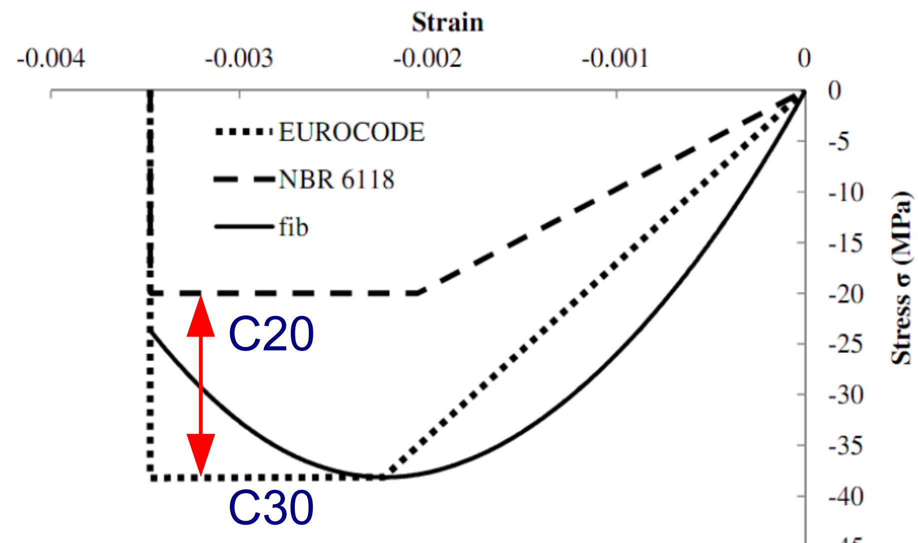
Applications – Planar Frame, Dynamics

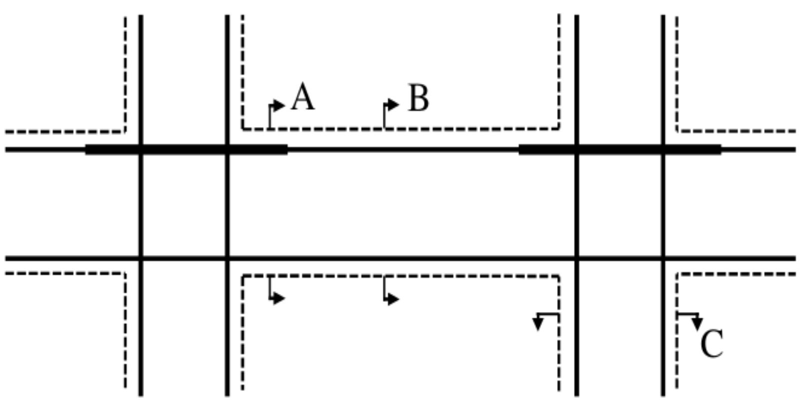
Eurocode 2 vs NBR6118



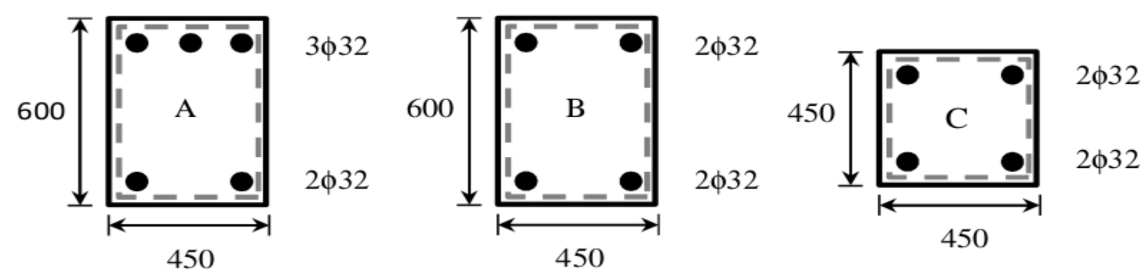
Steel quasi-static behavior



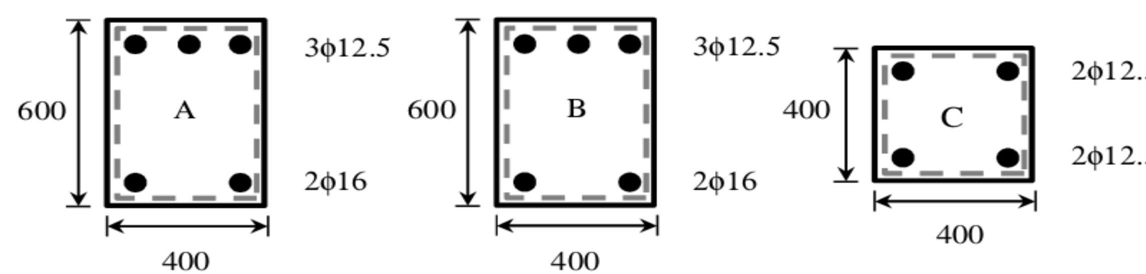
Concrete quasi-static behavior



Eurocode 2



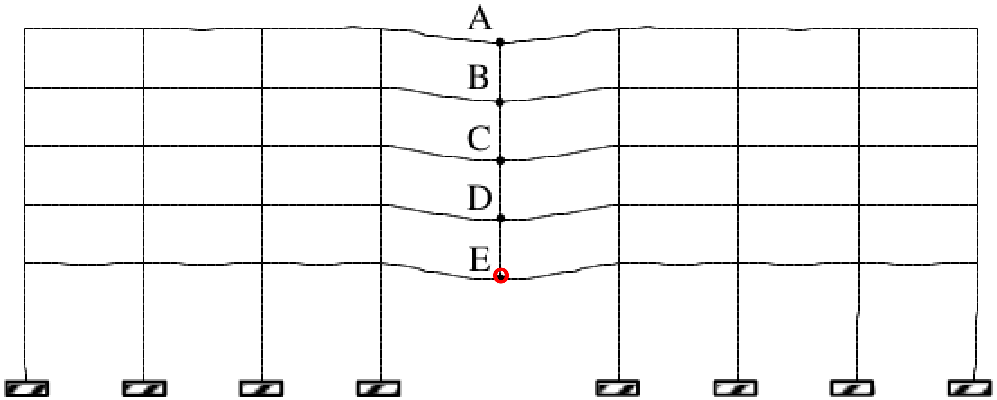
NBR6118



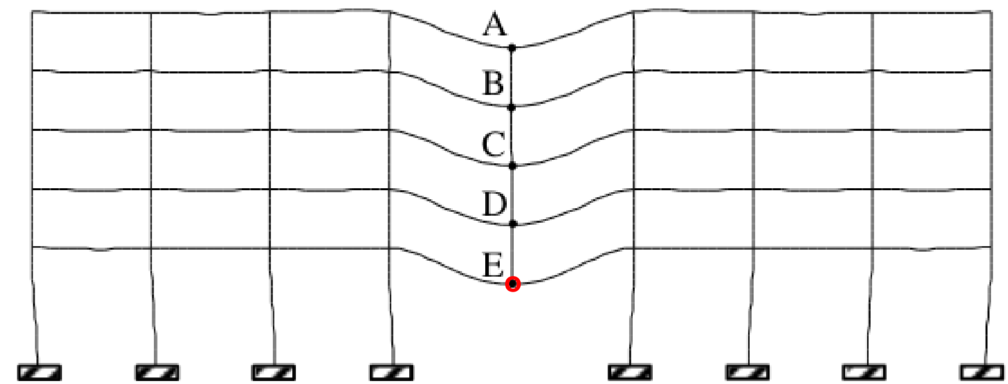
Applications – Planar Frame, Dynamics

Eurocode 2 vs NBR6118

Deformed configurations at t = 2 sec (displacements x 10)

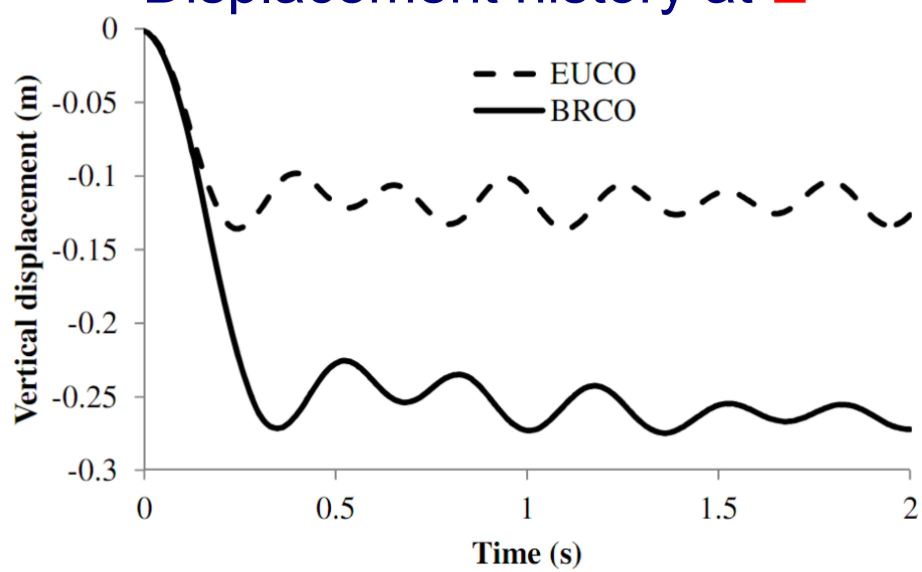


Eurocode 2



NBR6118

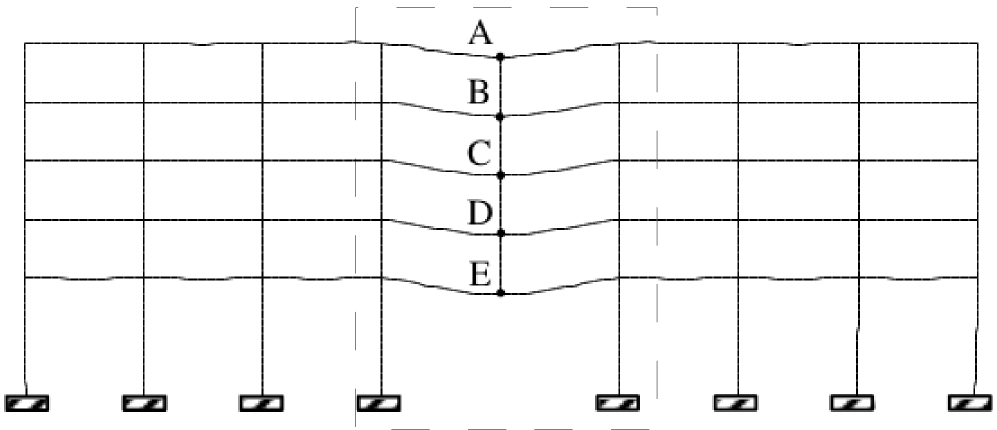
Displacement history at E



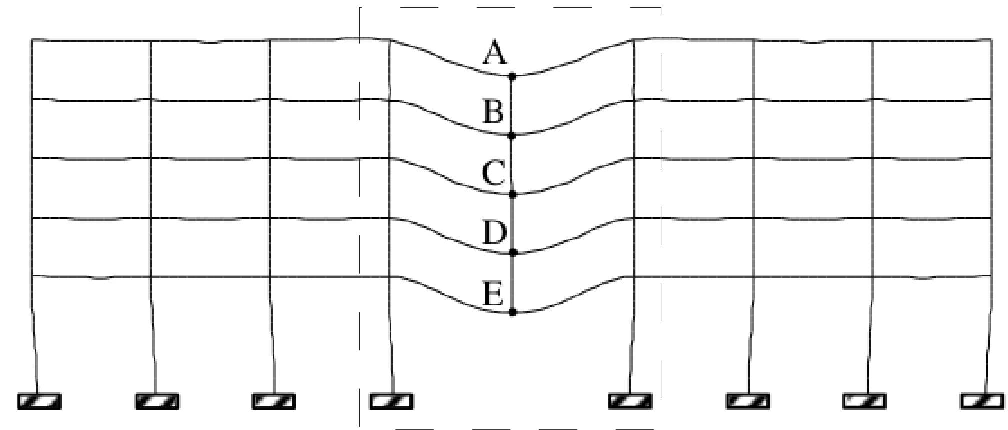
Applications – Planar Frame, Dynamics

Eurocode 2 vs NBR6118

Deformed configurations at t = 2 sec (displacements x 10)

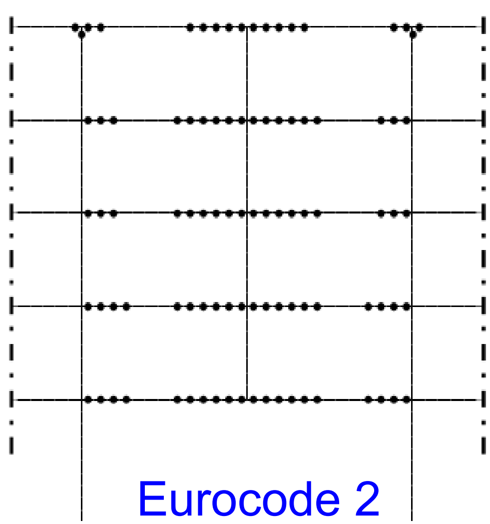


Eurocode 2

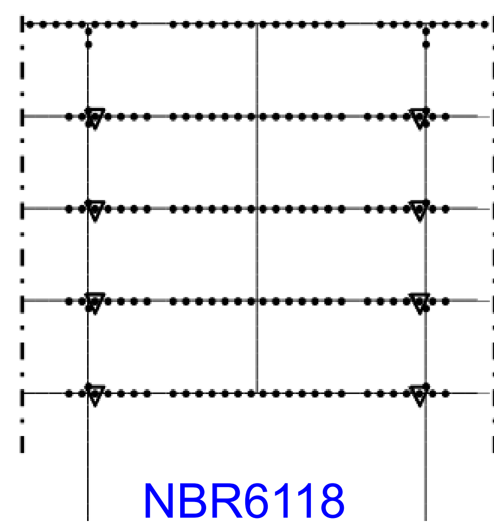


NBR6118

State of the sections at t = 2 sec



Eurocode 2



NBR6118

▽ Crushed concrete in less than 30% of the section

• Plastified steel

Applications – Planar Frame, Dynamics

Left column removal, bending moment evolution

Recommended literature

B.S. Iribarren, P. Berke, Ph. Bouillard, J. Vantomme, T.J. Massart, Investigation of the influence of design and material parameters in the progressive collapse analysis of RC structures, *Engineering Structures*, Vol. 33, page 2805-2820, 2011.

J.-M. Battini, Corotational beam elements in instability problems, *Ph.D. Thesis*, Royal Institute of Technology, Department of Mechanics, Stockholm, Sweden, 2002.

C.E.M. Oliveira, A.G. Marchis, P.Z. Berke, R.A.M. Silveira, T.J. Massart, Computational analysis of a RC planar frame using corotational multilayered beam FE, correlated to experimental results, In Proceedings of XXXIV CILAMCE, 13 pages, 2013