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Introduction to nonlinear finite element modeling

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4.0. Advanced subjects

Inspired and adapted from the ‘Nonlinear Modeling of Structures’ course of Prof. Thierry J. Massart at the ULB

ULB

Outline

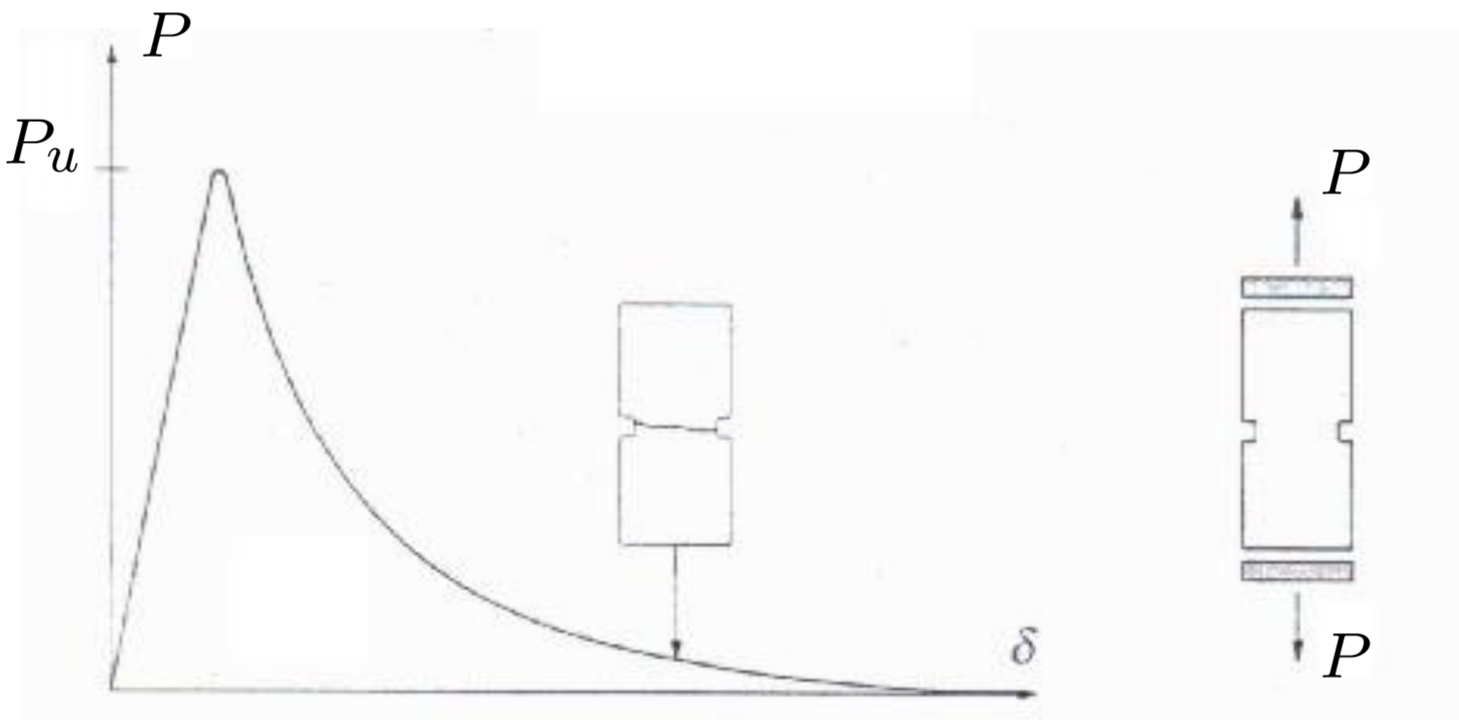
Treatment of softening behavior

Multiscale approach

Bifurcations

Origin of softening

Experimental curves : Force P vs displacement δ



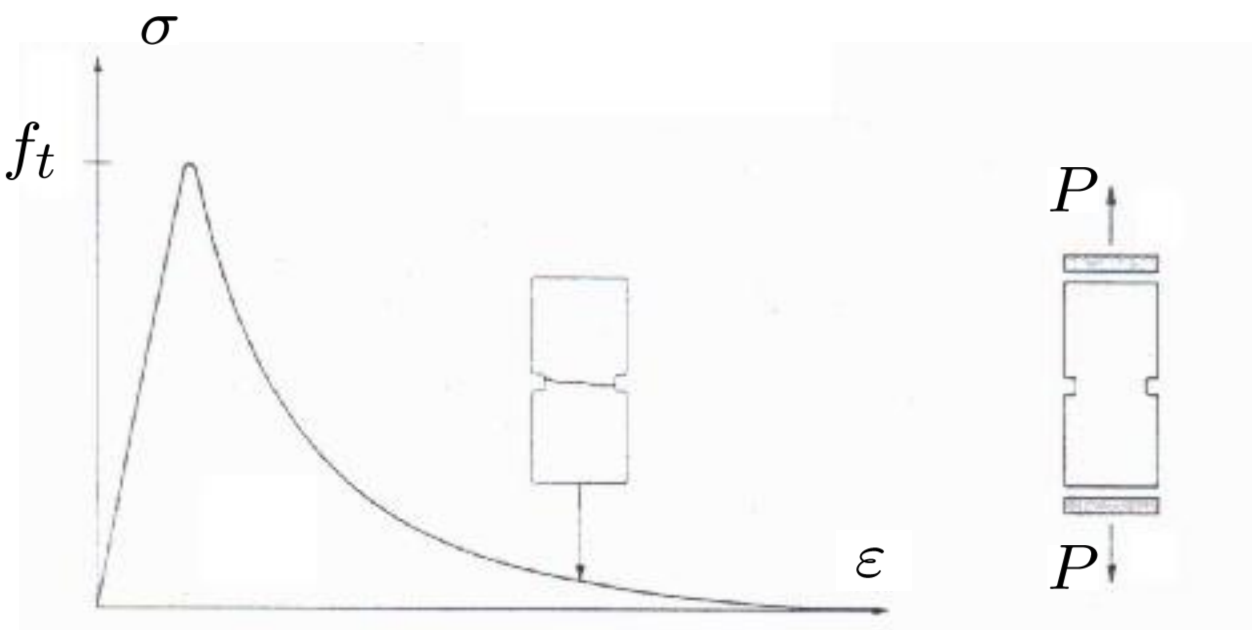
Lourenço, 1996

ATTENTION: This is a structural information, <u>not a material</u>, local <u>information</u>!

Softening in constitutive laws

Origin of softening

Such experimental curves are often translated into laws linking $\sigma - \varepsilon$ according to the following scheme



Lourenço, 1996

$$\sigma = \frac{P}{\Omega}$$

$$\varepsilon = \frac{\delta}{l_0}$$

Doing so, the information on the size of the cracking zone is lost

The structural behavior is <u>homogenised on the whole structural size</u>

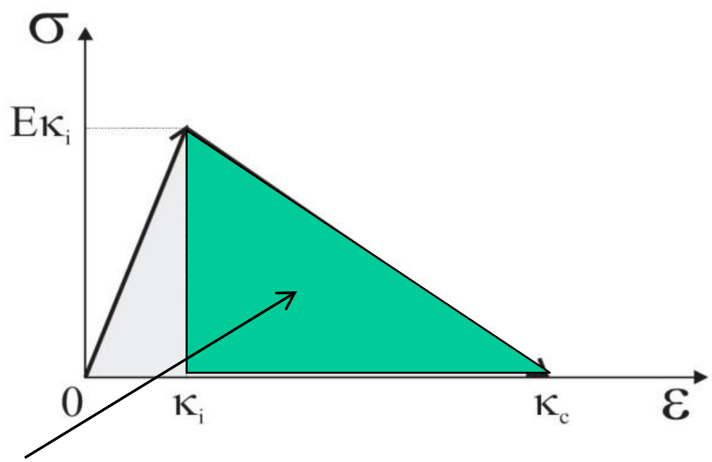
Softening in constitutive laws

Physics: energy is dissipated in the cracked volume

Crucial importance of the cracking width

Numerical challenge: capture proper energy dissipation

Crack width independent constitutive law?!



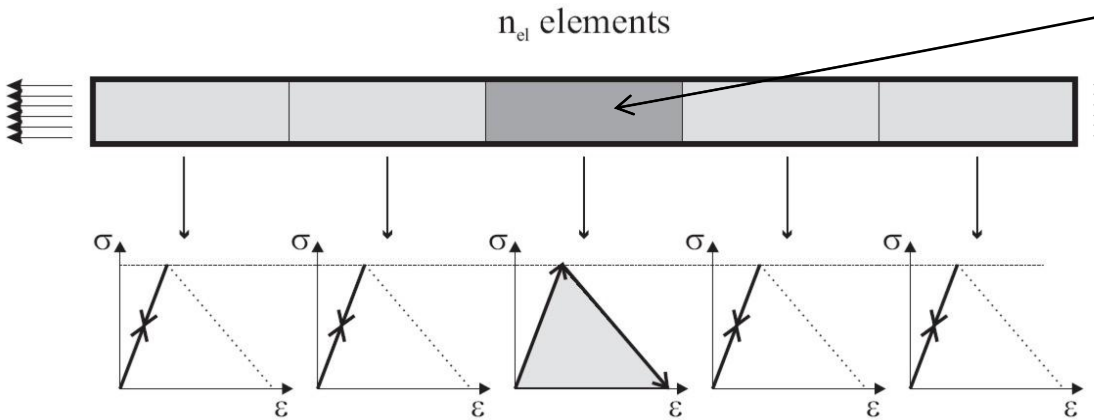
Work density required to create a unitary crack surface

Such a density has to be integrated on a finite size to yield a finite energy dissipation

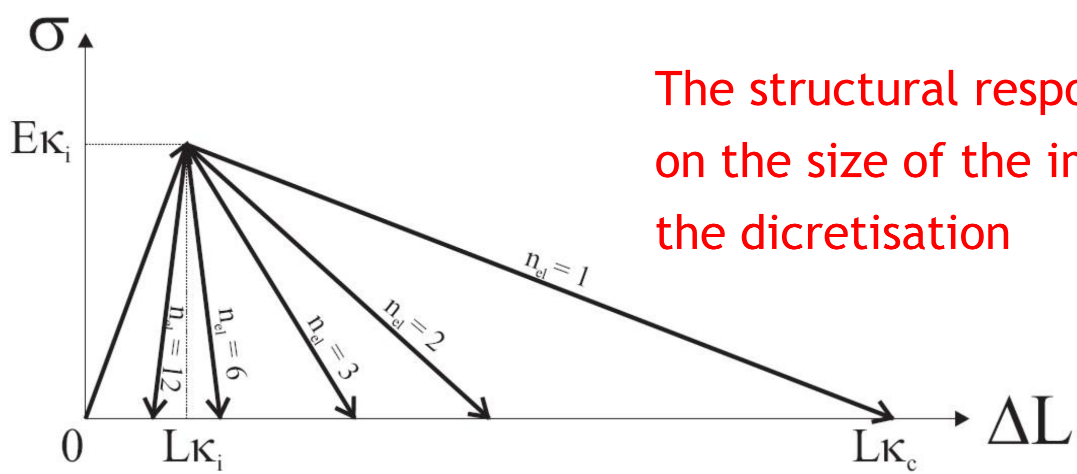
Pathological mesh dependency

Assume a finite elements discretisation with one imperfection

NO material localisation width is assumed (local law)

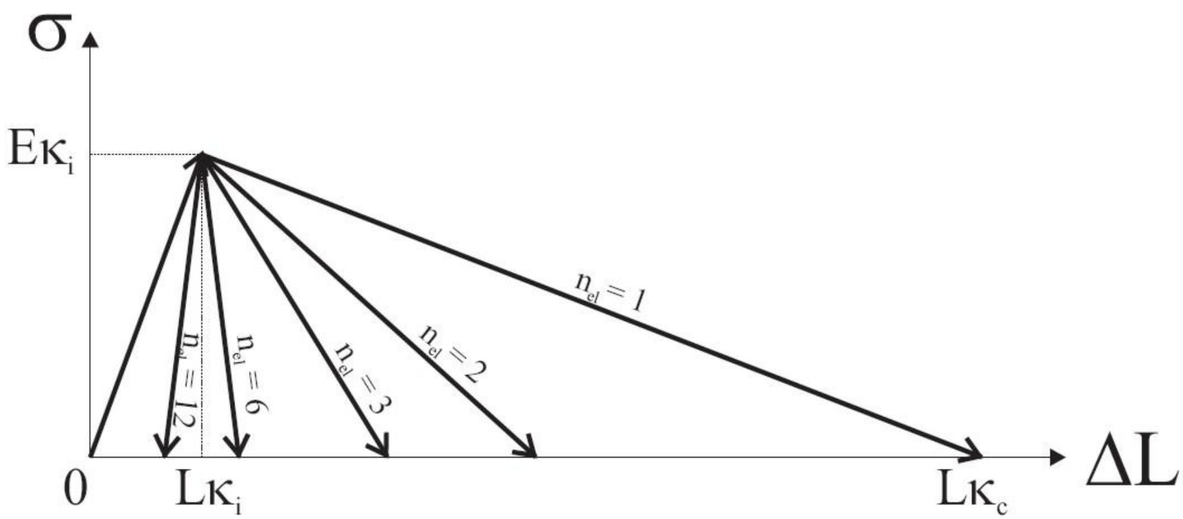


Bazant, 1998



The structural response depends on the size of the imperfection, i.e. the dicretisation

Pathological mesh dependency



Uniform stress → dissipation localises at the weak spot

The global structural dissipation only depends on the weak spot size

Global dissipation depends on the mesh refinement

Global dissipation occurs in a vanishing volume upon mesh refinement

There is convergence towards a vanishing dissipated energy

What are the solutions ???

With continuum descriptions

Adapt the material law to the mesh size (not very elegant)

Add a cracking width parameter in the formulation (complex)

With 'discrete' models

Model cracking with cohesive zones (zero thickness)

A priori positioned (less flexible) or positioned in the course of computation (complex)

Adapt the material response to the mesh

Assumption: localisation width h_c is a material property
fracture energy (per unit failure area): $G_F = h_c \gamma_F$

Numerical dissipated energy (with $h^{(e)}$ the element size)

$$G_{computational} = h^{(e)} \gamma_F^{(e)}$$

if $\gamma_F^{(e)} = \gamma_F$, and $h^{(e)} \neq h_c$ the dissipated energy is incorrect

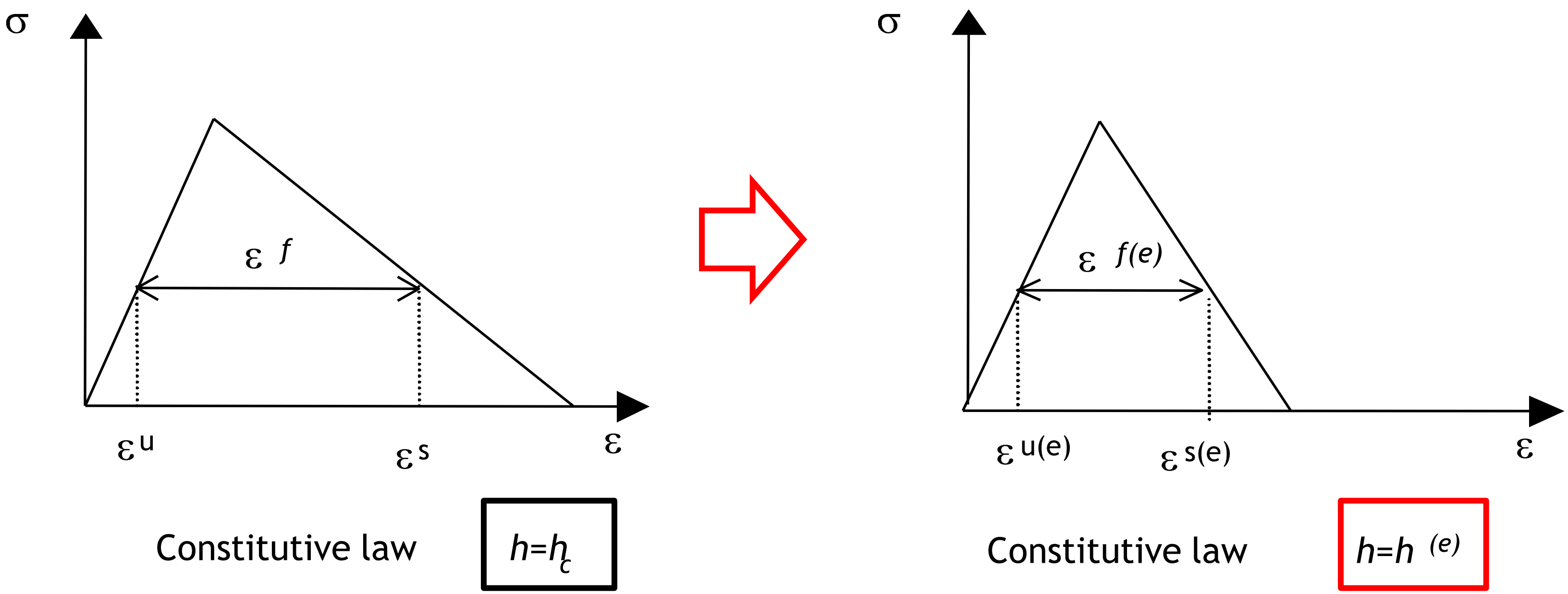
$\Downarrow$

Correct approximation of the GLOBAL fracture energy if:

$$\gamma_F^{(e)} = \frac{h_c}{h^{(e)}} \gamma_F$$

= scale the constitutive law as a function of the element size

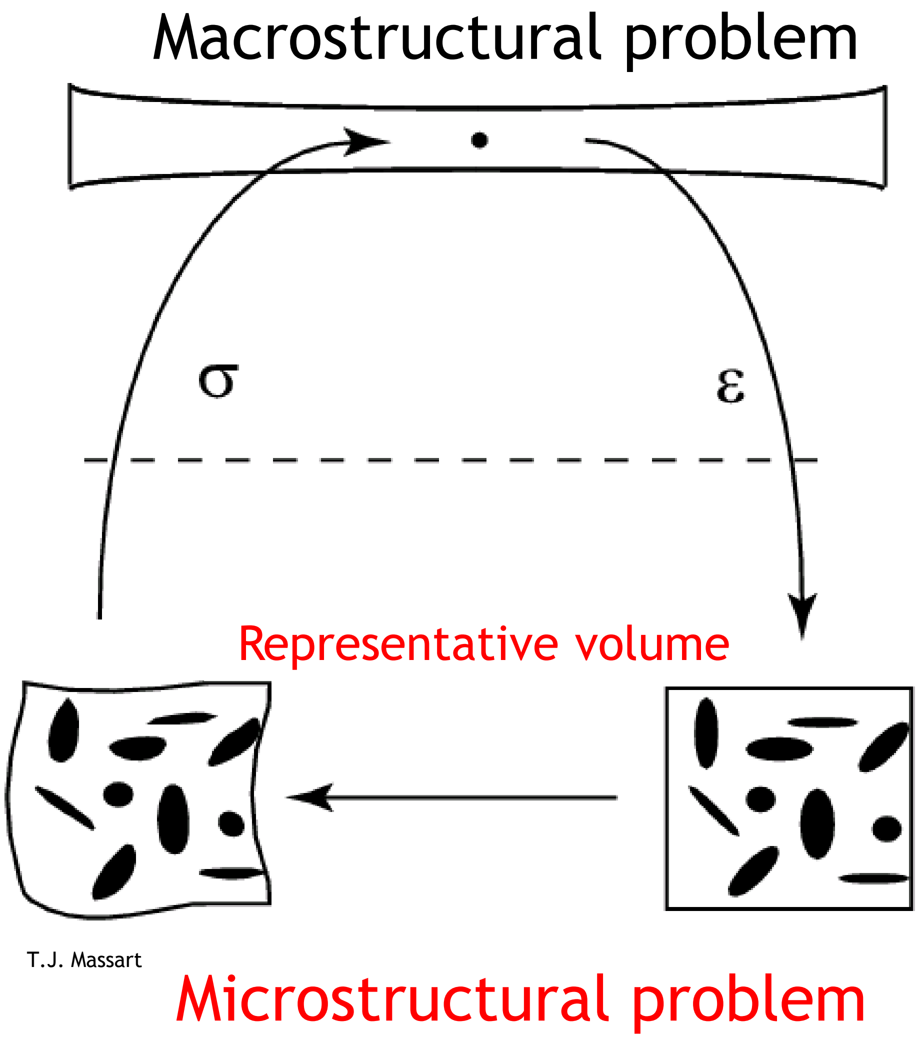
Adapt the material response to the mesh



This will restore the global energy dissipation… but at the expense of a ‘wrong’ local response

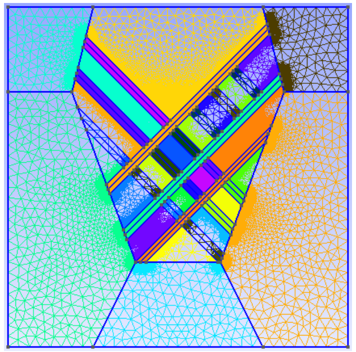
This does NOT solve the root of the problem (loss of ellipticity)

Multiscale approach



Macro behavior computed numerically

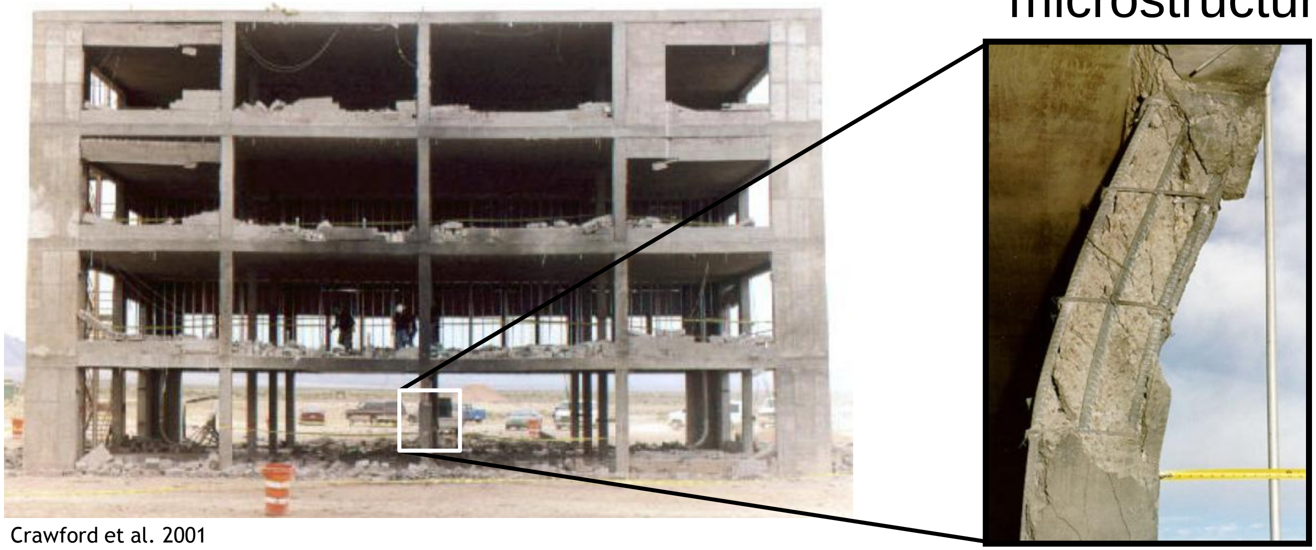
- Constitutive laws for the constituents
- Complex material microstructure



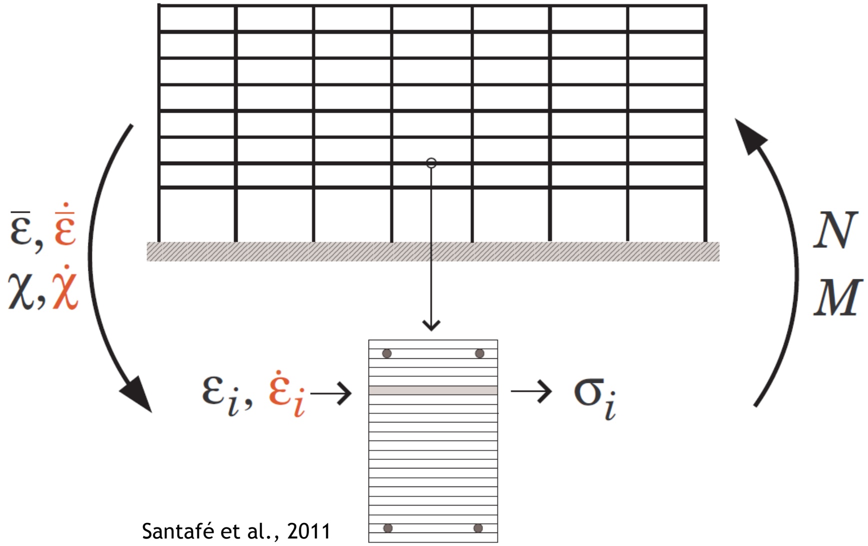
Layered beam example

Structure

Material microstructure

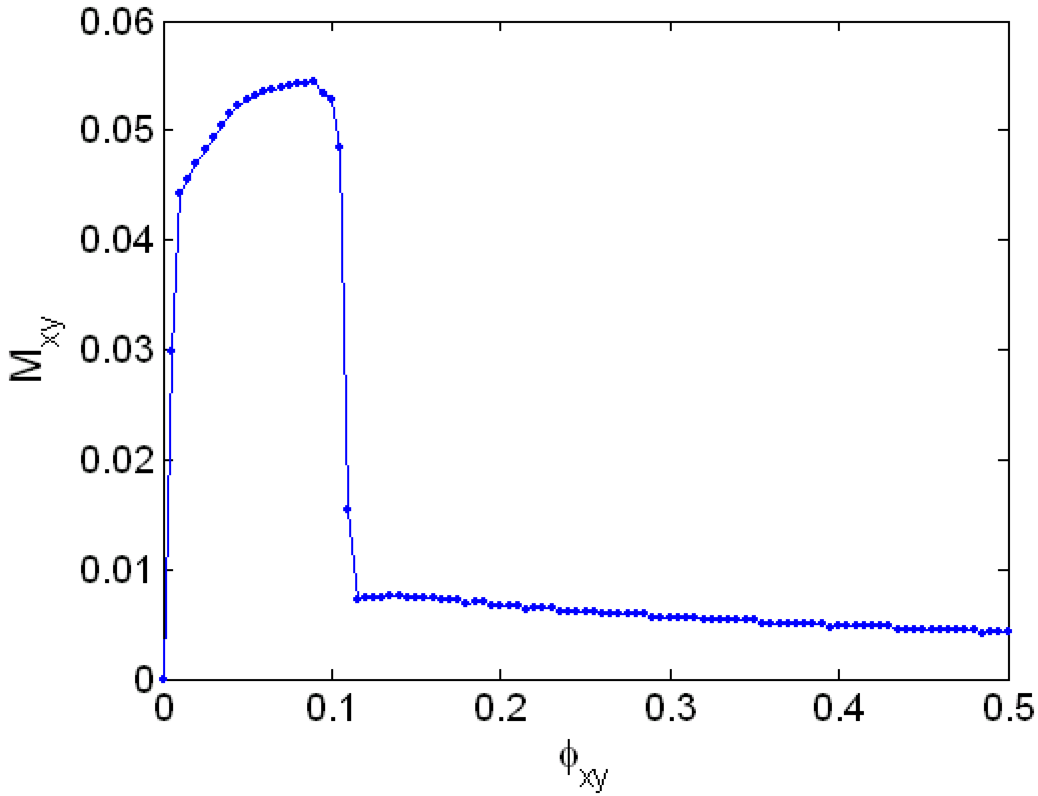


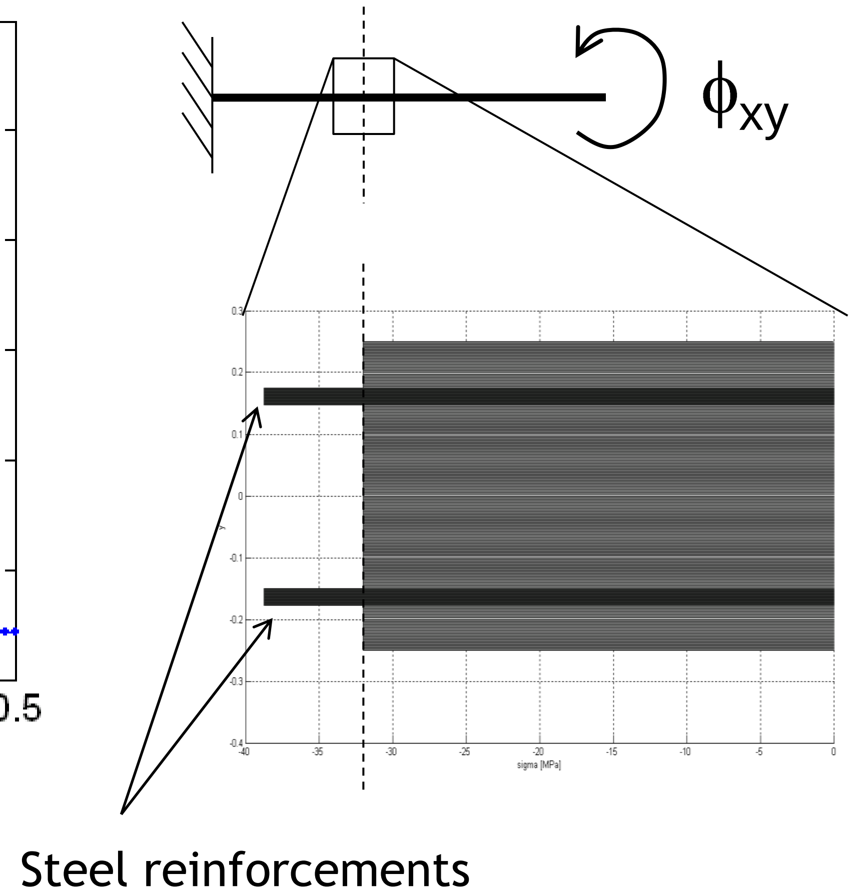
Crawford et al. 2001



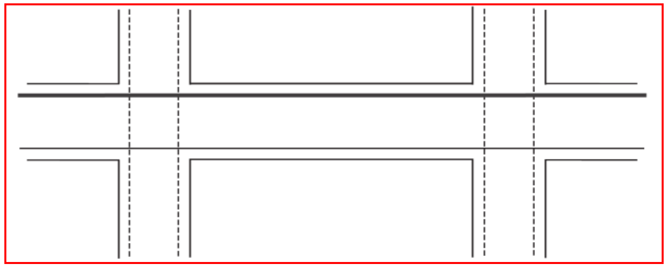
Santafé et al., 2011

Beam bending

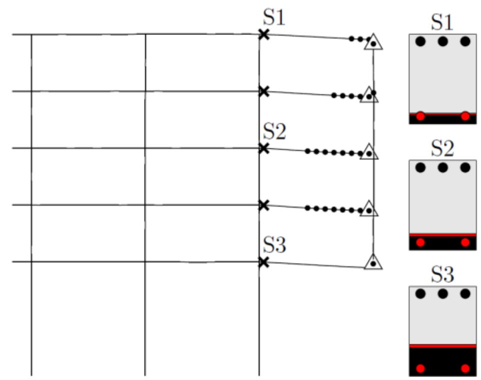




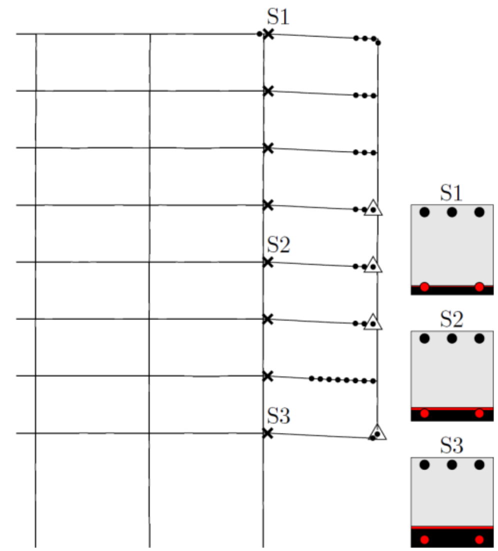
Application: progressive collapse



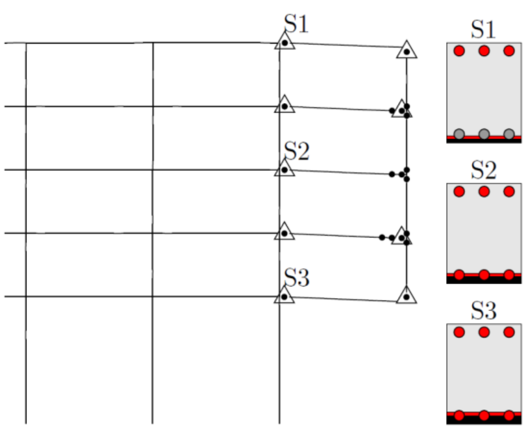
Collapse!



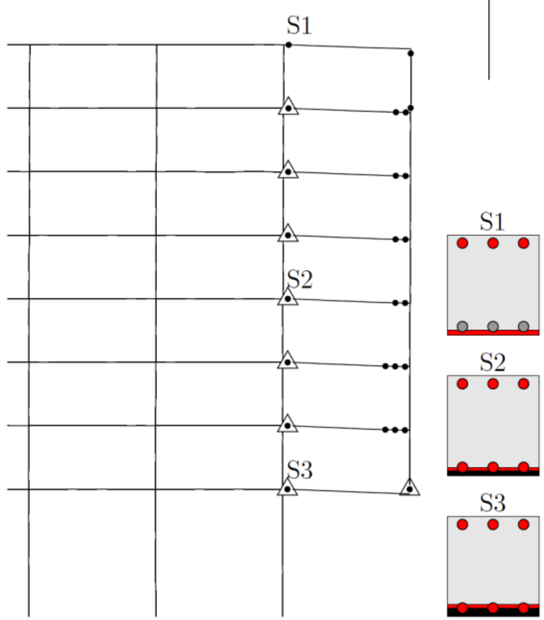
[after 410 ms]



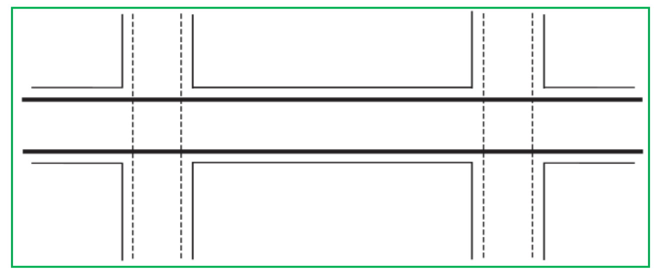
[after 360 ms]



[after 2000 ms]



[after 2000 ms]



OK!

Santafé et al., 2011

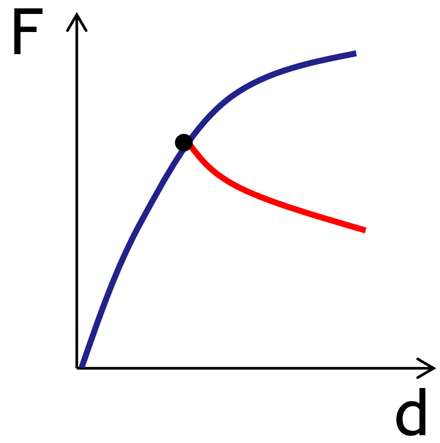
Bifurcation point

Points as from which several solutions exist for equilibrium

- Bifurcations ⟵⟶ non-linearity (material or geometrical)
- Example: Buckling

At least 2 equilibrium paths (= curves) exist in this point

- Path without perturbation = **fundamental path**
- Other paths obtained by **perturbation** = **alternative paths**
- Example: Eulerian buckling of beams
  - Fundamental path = unlimited compression
  - Bifurcation point = buckling limit point
  - Alternative paths = sinusoidal solutions



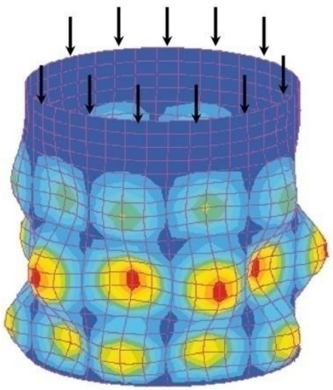
Bifurcation points

Importance of a correct treatment

- All solutions satisfy the equilibrium equations
- The real solution is the one that minimises the supplied work (most critical)
- The fundamental solution is rarely the most critical
- Risk of overestimation of limit loads

Example: elastic instabilities in shells

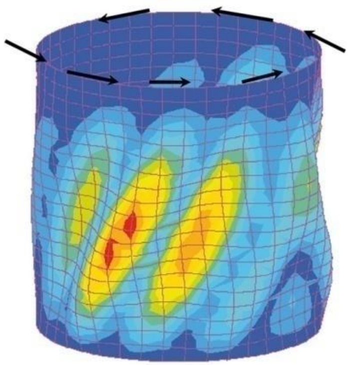
Compression



[www.samcef.com]

[http://www.mech.uwa.edu.au/DANotes/buckling/intro/intro.html]

Torsion



[www.samcef.com]

[http://www.dae.gov.in]

Bifurcation detection

If two equilibrium solutions exist

$$\begin{cases} {}^{t}[K_t]\ {}^{t+\Delta t}\{\Delta q\} = \Delta\omega \left\{ f_{ext}^{(u)} \right\} \\ {}^{t}[K_t]\ {}^{t+\Delta t}\{\Delta q^*\} = \Delta\omega \left\{ f_{ext}^{(u)} \right\} \end{cases}$$

$${}^{t}[K_t]\left({}^{t+\Delta t}\{\Delta q\} - {}^{t+\Delta t}\{\Delta q^*\} \right) = 0$$

Two different solutions are possible if

$$\det\left({}^{t}[K_t] \right) = 0 \qquad \lambda_{min}^{{}^{t}[K_t]} = 0$$

This requires the correct tangent to be used

(i.e. modified Newton method not viable)

Practical implications

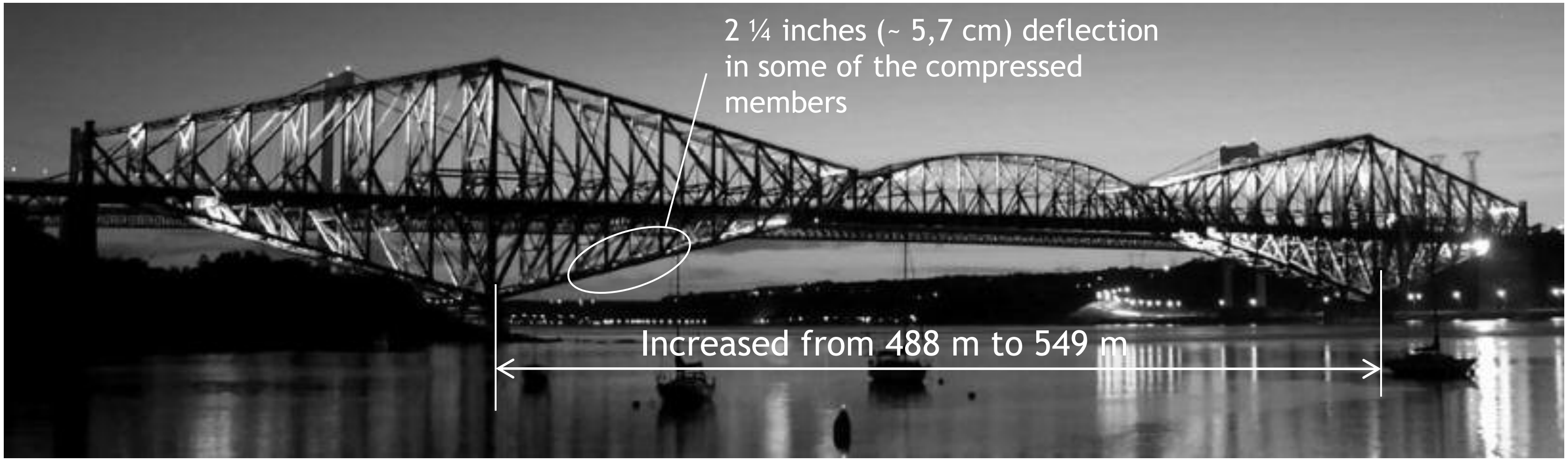
For a nonlinear computation with a risk of bifurcation

- Detect at each converged step the presence of a bifurcation
- Any negative eigenvalue indicates a potential bifurcation

To determine the physically realised response

- Start from the converged situation corresponding to the detected bifurcation
- Perturbation of solution with eigenvector associated with the vanishing eigenvalue
- The most critical solution minimises the variation of supplied energy
- ALL of the solutions have to be examined to identify the most critical one

Global structural instability



Pont de Québec 1904-1907 (ambition: the longest bridge at the time)

Collapse in 15 s due to the buckling of compressed elements.

[http://www.civeng.carleton.ca/ECL/reports/ECL270/Disaster.html]
[http://www.bernd-nebel.de/bruecken/4_desaster/quebec/quebec.html]