UFOP, Departamento de Engenharia Civil
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Nonlinear Multi-Scale Finite Element Modeling of the Progressive Collapse of Reinforced Concrete Structures

Lecture 3: Computational plasticity

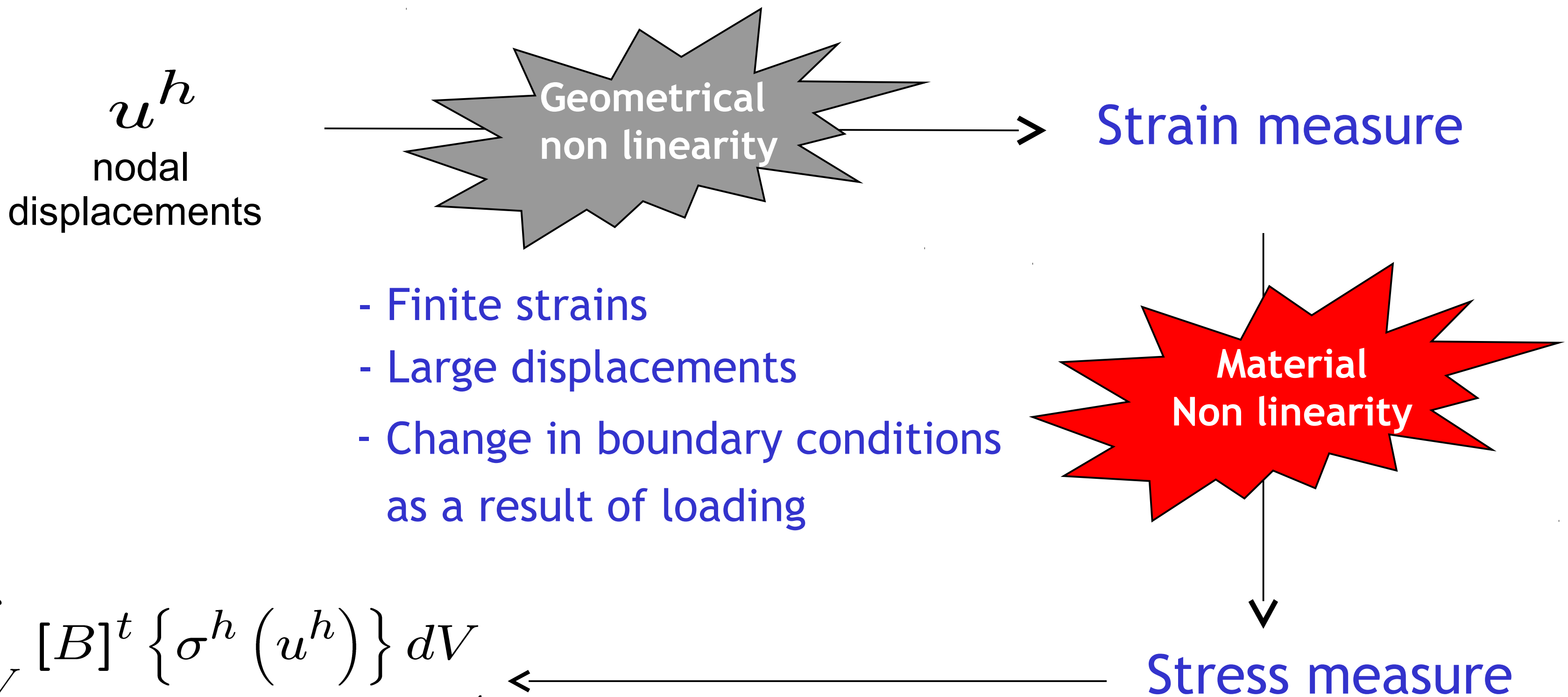
Péter Z. Berke

Visiting Professor
Departamento de Engenharia Metalúrgica e de Materiais
Universidade Federal do Ceará, Bloco 729
Scientific Collaborator
Building, Architecture and Town Planning Dept. (BATir)
Université Libre de Bruxelles (ULB), Brussels, Belgium

Inspired and adapted from the 'Nonlinear Modeling of Structures' course of Prof. Thierry J. Massart at the ULB

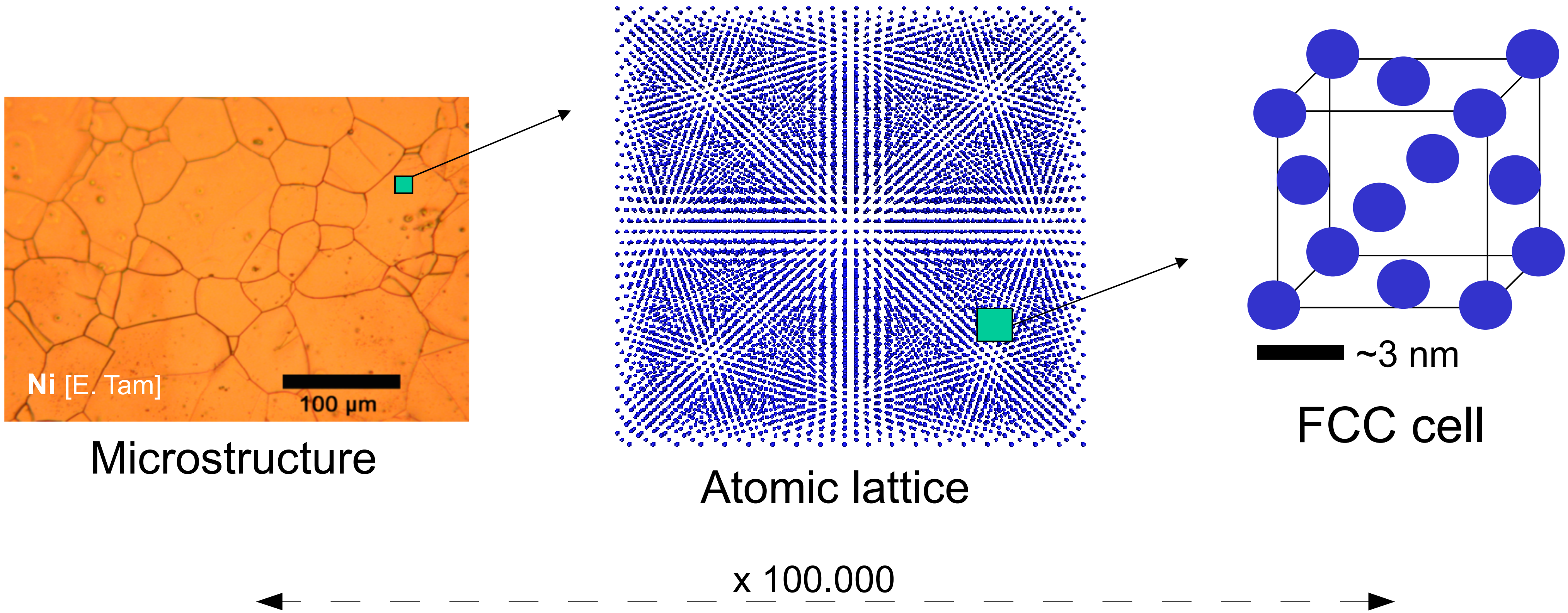
Sources of nonlinearity

Non proportionnality between applied forces and displacements



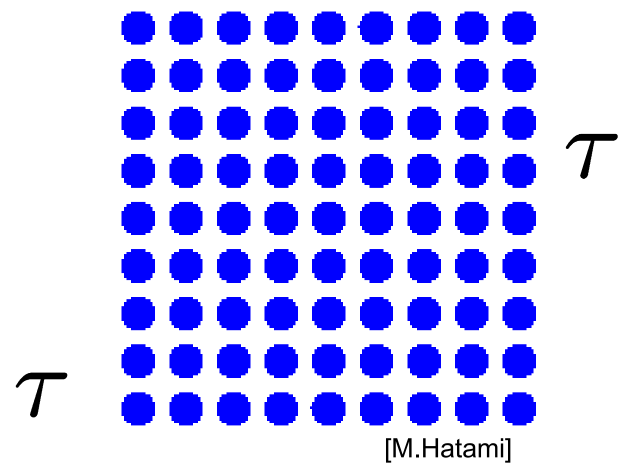
$$\underbrace{\int_V [B]^t \left\{\sigma^h \left(u^h\right)\right\} dV}_{\{f_{int}\}}$$

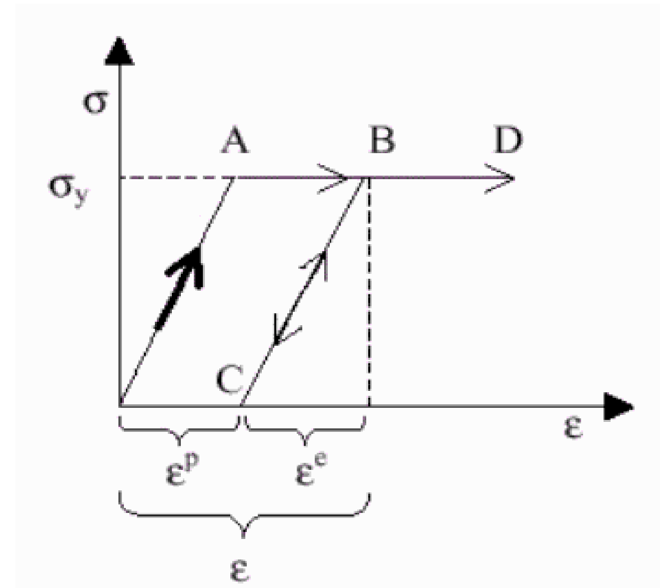
Micromechanical origins of plasticity



Permanent deformation induced by glide on crystallographic planes

Micromechanical origins of plasticity





Perfect plasticity: glide of cristallographic planes under constant stress

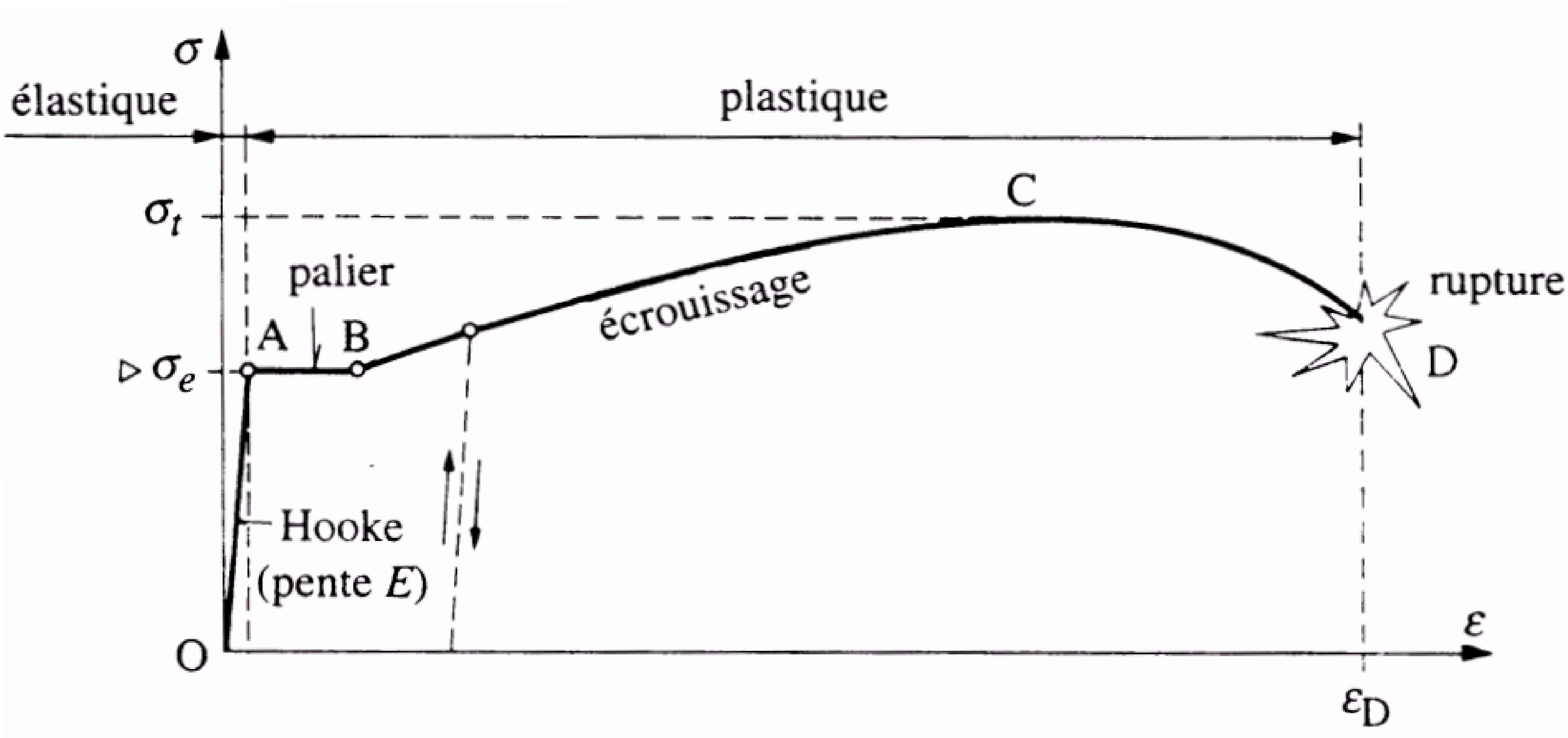
Irreversibility manifests itself through **permanent strains**

Reversible stress states (without permanent strains) are limited by the stress level σ_y (states $\sigma > \sigma_y$ are impossible with permanent strains unchanged)

Reversibility of a state change **depends the stress**

Hardening plastic model (macroscale)

- Glide of cristallographic planes impeded by dislocations
- Increase of σ needed to produce further plastic strains



B – C : extension of the set of admissible stress states without further permanent strain increase

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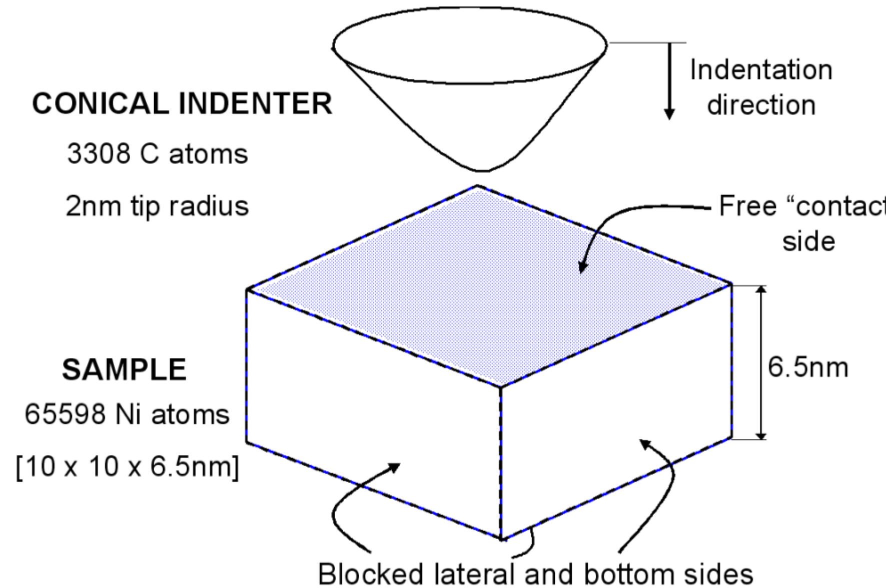
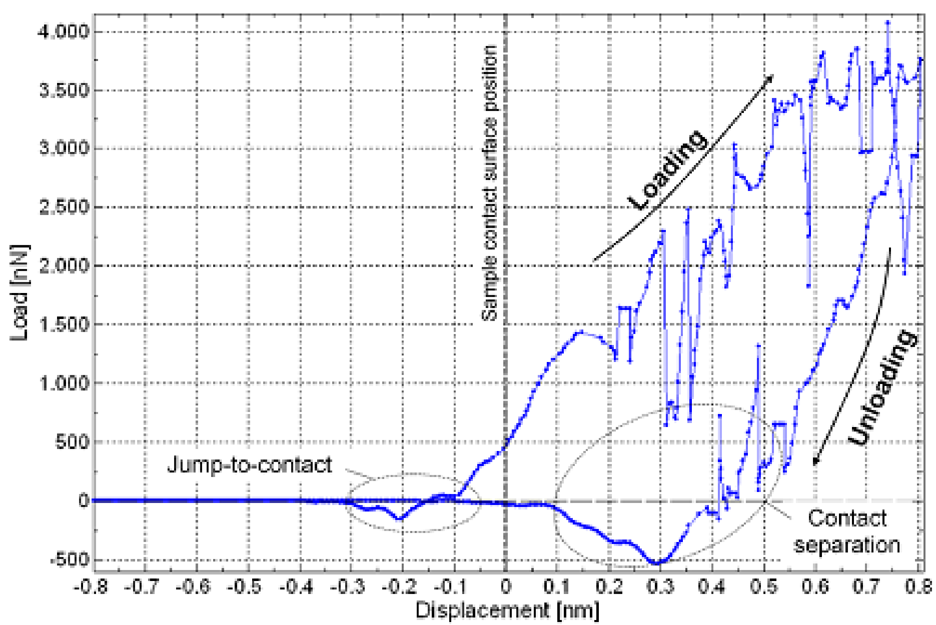
Atomic scale model of plastic indentation





Interaction of dislocations

\+

Contact evolution

=

Increasing stress necessary to induce further penetration

Course planning

- Phenomenological material model of constituents of RC
- Uniaxial and multiaxial plastic constraints
  - Constitutive law in the structural solution procedure
  - Implicit nature of the problem at Gauss points
  - Return Mapping (uniaxial case)
  - Loops in solution procedure with plasticity
- Beam-column plastic frame example
  - Elastic vs. plastic modeling

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From material to structural behavior

Structure

[Experimental blast test, Crawford et al. 2001]

Material microstructure

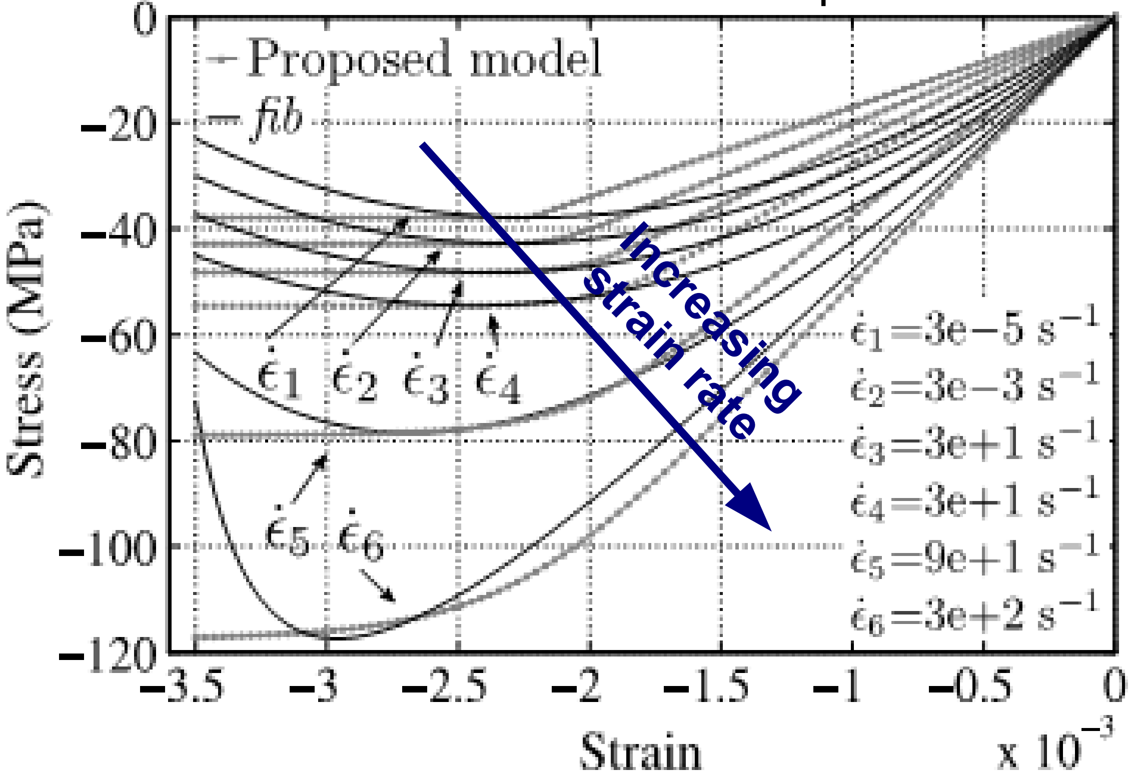
Material behavior

concrete

steel

Concrete material behavior

Concrete behavior in compression



[The International Federation for Structural Concrete *(fib)*]

- E strain rate dependent

$$\frac{E_c}{E_{c,st}} = \left(\frac{\dot\epsilon}{\dot\epsilon_{st}}\right)^{0.026}$$

- Plasticity strain rate dependent

$$\frac{f_c}{f_{c,st}} = \begin{cases} \left(\frac{\dot\epsilon}{\dot\epsilon_{st}}\right)^{1.026\alpha_s} & \text{for } \dot\epsilon \le 30\ \text{s}^{-1} \\ \gamma_s \left(\frac{\dot\epsilon}{\dot\epsilon_{st}}\right)^{1/3} & \text{for } \dot\epsilon > 30\ \text{s}^{-1} \end{cases}$$

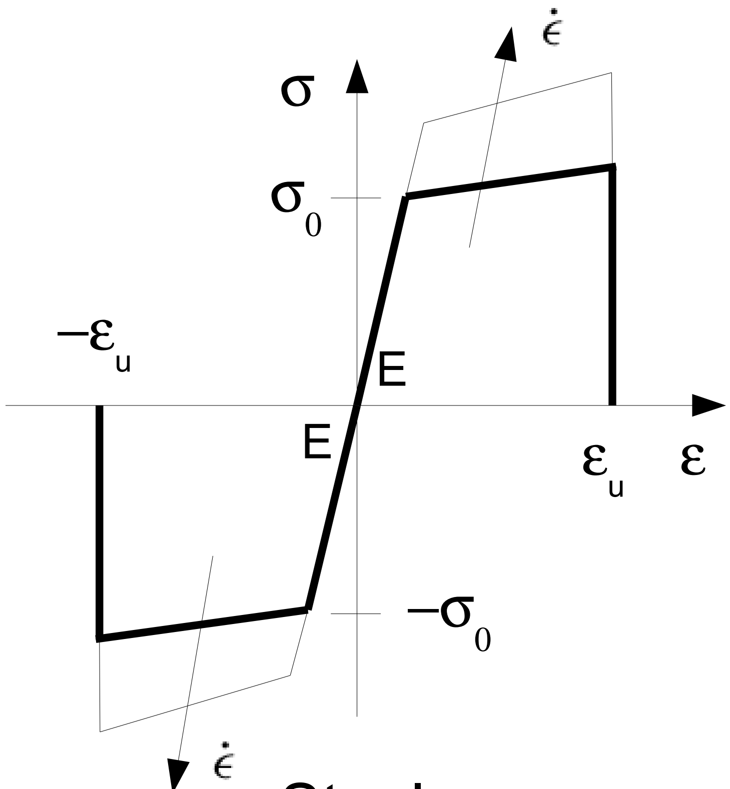
Perzyna viscoplastic model

$$\dot\epsilon^{vp} = \frac{1}{\eta}\left\langle\frac{f}{\bar\sigma_0}\right\rangle^N \frac{\mathrm{d}f}{\mathrm{d}\sigma}$$

[1] B.S. Iribarran et al., Engng. Struct., 33, 2805-2820 (2011)

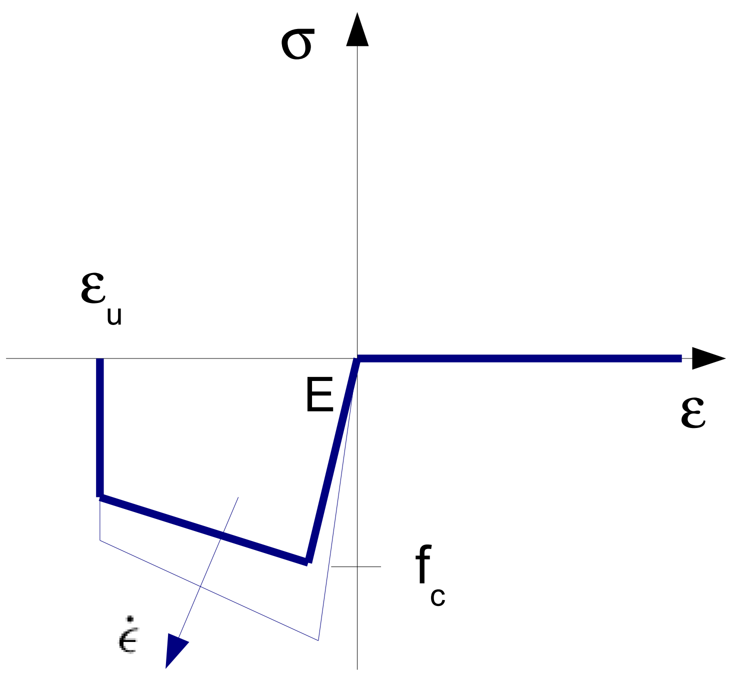
1D Constitutive models of constituents

1D laws fitted to experimental data



Steel

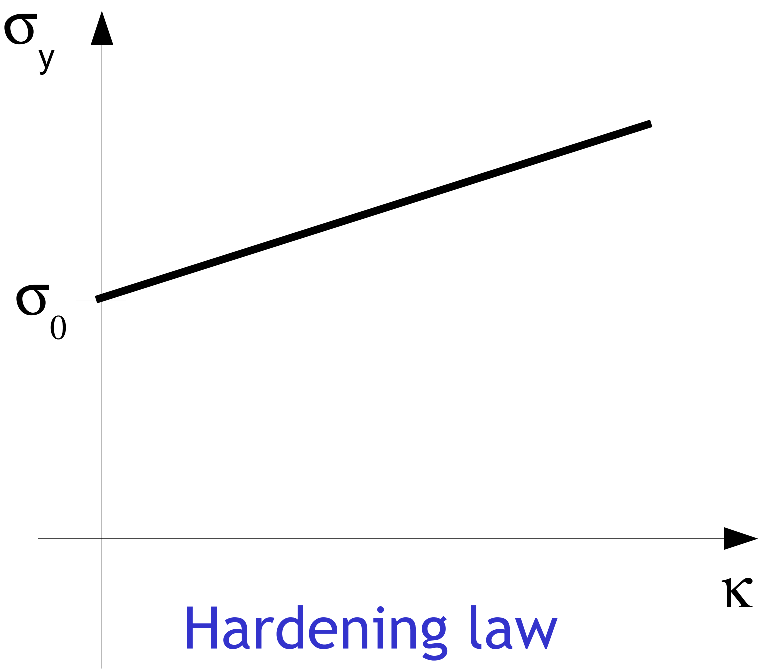
- Plasticity strain rate dependent
- Ultimate cutoff strain



Concrete

- E strain rate dependent
- Plasticity strain rate dependent
- No resistance to traction
- Ultimate cutoff strain

Uniaxial plasticity



Hardening law

Hardening parameter κ

History parameter representing the cumulated effect of plastic dissipation

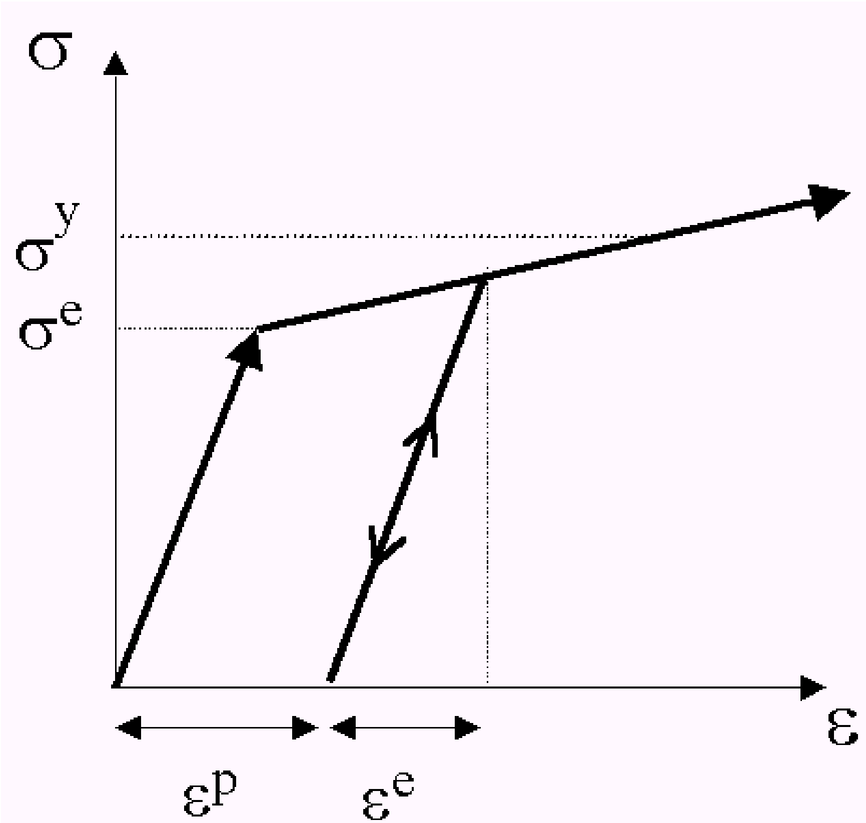
Current yield strength σ_y

Stress that has to be exceeded to induce further plastic straining

Hardening

If the plastic strain increases the admissible domain expands (if $\sigma_y(\kappa)$ is an increasing function)

Uniaxial plasticity



- Hardening behavior
- Strain rate independent

Strain partition

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

Plasticity criterion in stress space

$f^p = \sigma - \sigma_y(\kappa) = 0 \rightarrow$ Increase of ε^p

$f^p = \sigma - \sigma_y(\kappa) < 0 \rightarrow$ Elastic behaviour

$f^p = \sigma - \sigma_y(\kappa) > 0 \rightarrow$ Non admissible states

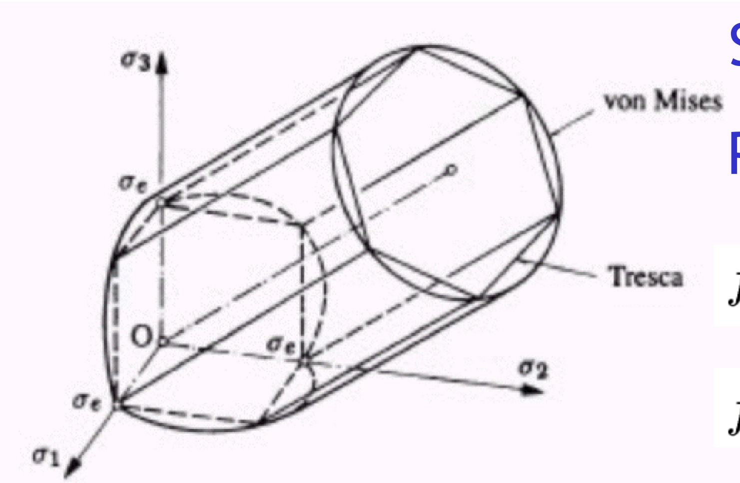
Constitutive relation

$$\sigma = E\varepsilon^e = E(\varepsilon - \varepsilon^p)$$

Consistency condition

The point representing the stress state in the stress space has to remain on the reversible domain when plastic strains are increasing

Multiaxial plasticity – yield surface



Strain partition $\varepsilon = \varepsilon^e + \varepsilon^p$

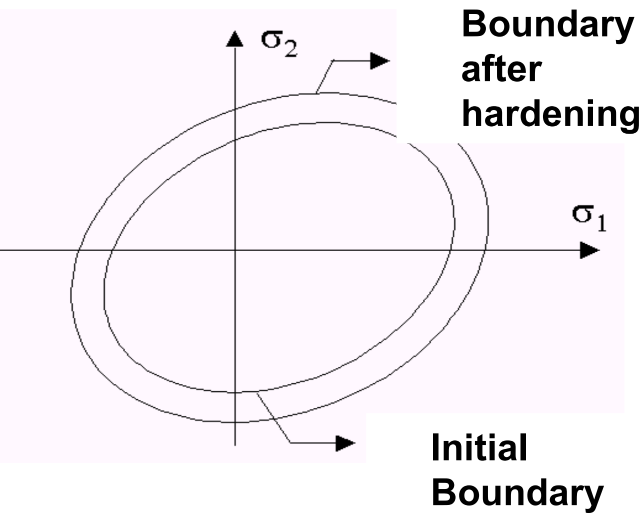
Plasticity criterion in stress space (isotropic)

principal stresses

$f^p = \sigma_{eq}(\sigma_1, \sigma_2, \sigma_3) - \sigma_y(\kappa) = 0$ → Increase of ε^p

$f^p = \sigma_{eq}(\sigma_1, \sigma_2, \sigma_3) - \sigma_y(\kappa) < 0$ → Elastic behaviour

$f^p = \sigma_{eq}(\sigma_1, \sigma_2, \sigma_3) - \sigma_y(\kappa) > 0$ → Non admissible states



Constitutive relation

$$\{\sigma\} = [H]\{\varepsilon^e\} = [H](\{\varepsilon\} - \{\varepsilon^p\})$$

Consistency condition

'Direction' of plastic strains

$$\{d\varepsilon^p\} = d\lambda \left\{ \frac{\partial g^p}{\partial \sigma} \right\}$$ $g^p = f^p$ most often chosen for metals

Structure of a NL FE code

Define a set of successive loading states $F_{ext,n}$

Loop on the loading states (steps or increments)

Formulate the problem for the step $n \rightarrow n+1$

Find q_{n+1} such that $F_{int}(q_{n+1}) - F_{ext,n+1} = 0$

With as first approximation $q_{n+1}^{(0)} = q_n$

Iterate until a precision threshold is reached with

$$^{t+\Delta t}\{f_{int}\}^{(i)} = \int_{V_e} [B]\ ^{t+\Delta t}\{\sigma\}^{(i)}\ \mathrm{dV_e}$$

$$q_{n+1}^{(k+1)} = q_{n+1}^{(k)} - \left(\frac{\partial F_{int}}{\partial q}\right)^{-1}_{q_{n+1}^{(k)}} \left(F_{ext,n+1} - F_{int,n}^{(k)}\right)$$

K_T

End of increment

Constitutive law in solution procedure

Structural tangent stiffness at iteration (i)

$$\underbrace{\left[\frac{\partial\ {}^{t+\Delta t}\{f_{int}\}^{(i-1)}}{\partial q}\right]}_{\left[K_t\left({}^{t+\Delta t}\{q\}^{(i-1)}\right)\right]}\ {}^{t+\Delta t}\{\delta q\}^{(i)} \simeq {}^{t+\Delta t}\{f_{ext}\} - {}^{t+\Delta t}\{f_{int}\}^{(i-1)}$$

$${}^{t+\Delta t}[K_t]^{(i-1)} = \sum_e \left(\int_{V_e} [B]^T\ {}^{t+\Delta t}[\mathbf{L}]^{(i-1)} [B]\, \mathrm{dV_e}\right)$$

with $\{\Delta\sigma\} = \underbrace{{}^{t}\left[\frac{\partial\sigma}{\partial\varepsilon}\right]}_{{}^{t}[\mathbf{L}]}\{\Delta\varepsilon\}$

Internal force computation at iteration (i)

$${}^{t+\Delta t}\{q\}^{(i)} \rightarrow {}^{t+\Delta t}\{\varepsilon\}^{(i)} \rightarrow {}^{t+\Delta t}\{\sigma\}^{(i)}$$

$$\rightarrow \quad {}^{t+\Delta t}\{f_{int}\}^{(i)} = \int_v [B]\ {}^{t+\Delta t}\{\sigma\}^{(i)}\, dv$$

Computation of internal forces

$$^{t+\Delta t}\{f_{int}\}^{(i)} = \int_{V_e} [B] \ ^{t+\Delta t}\{\sigma\}^{(i)} \ \mathrm{dV_e}$$

Stress update at iteration (i) at each Gauss point $\sigma(\varepsilon)$

Known values $^{t}\{\sigma\}, \ ^{t}\{\varepsilon\}, \ ^{t}\{\varepsilon^p\}, \ ^{t}\kappa$ (last converged configuration)

Strain update $^{t+\Delta t}\{\Delta\varepsilon\}^{(i)} = [B] \ ^{t+\Delta t}\{\Delta q\}^{(i)}$

To be determined $^{t+\Delta t}\{\sigma\}, \ ^{t+\Delta t}\{\varepsilon^p\}, \ ^{t+\Delta t}\kappa$

Implicit nature of the problem in each Gauss point

The strain update is known $^{t+\Delta t}\epsilon = ^{t}\epsilon + ^{t+\Delta t}\Delta\epsilon$

But the reversibility criterion is expressed in the stress space

$$\sigma_{eq}\left(^{t+\Delta t}\{\sigma\}^{(i)}\right) - \sigma_y\left(^{t+\Delta t}\kappa^{(i)}\right) = 0$$

where $^{t+\Delta t}\{\sigma\}^{(i)}$ and $^{t+\Delta t}\kappa^{(i)}$ are unknown

Implicit problem at Gauss points

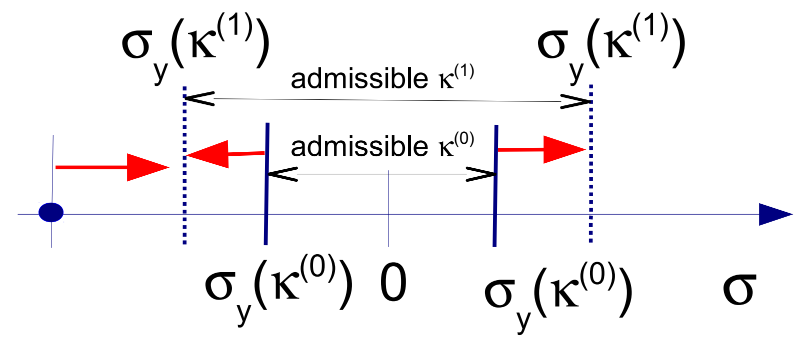
The strain update is known $^{t+\Delta t}\epsilon = ^{t}\epsilon + ^{t+\Delta t}\Delta\epsilon$

$$^{t+\Delta t}\Delta\epsilon = ^{t+\Delta t}\Delta\epsilon^{e\,(i)} + ^{t+\Delta t}\Delta\epsilon^{p\,(i)}$$

$$^{t+\Delta t}\Delta\sigma^{(i)} = E\left(^{t+\Delta t}\Delta\epsilon - ^{t+\Delta t}\Delta\epsilon^{p\,(i)}\right)$$

$$^{t+\Delta t}\sigma^{(i)} = ^{t}\sigma + ^{t+\Delta t}\Delta\sigma^{(i)}$$

$$^{t+\Delta t}\kappa^{(i)} = ^{t}\kappa + ^{t+\Delta t}\Delta\kappa^{(i)}$$



if $f_p\left(^{t+\Delta t}\sigma^{(i)},\ ^{t+\Delta t}\kappa^{(i)}\right) > 0$

- decrease $^{t+\Delta t}\sigma^{(i)}$ by increasing plastic strain $^{t+\Delta t}\Delta\kappa^{(i)}$

simultaneously!

- increase in $^{t+\Delta t}\Delta\kappa^{(i)}$ inflates $f_p\left(^{t+\Delta t}\sigma^{(i)},\ ^{t+\Delta t}\kappa^{(i)}\right)$

Solution - Return Mapping (1D case)

At each Gauss point

$$\sigma^{(k)} = \sigma_c + \mathrm{E}\,(\Delta\epsilon - \Delta\epsilon_p^{(k)})$$

$$\sigma_{tr} = \sigma_c + \mathrm{E}\,\Delta\epsilon$$ trial stress – elastic increment assumption

if $f_p\left(\sigma_{tr},\, {}^t\kappa\right) > 0$

$$\sigma^{(k)} = \sigma_{tr} - \mathrm{E}\,\Delta\epsilon_p^{(k)}$$

$$\frac{\sigma^{(k)} - \sigma_{tr}}{\mathrm{E}} + \Delta\epsilon_p^{(k)} = 0$$

solve using Newton-Raphson

$$\begin{cases} \dfrac{\sigma^{(k)} - \sigma_{tr}}{\mathrm{E}} + \Delta\epsilon_p^{(k)} = 0 \\ f(\sigma^{(k)},\, \kappa^{(k)}) = 0 \end{cases}$$

$$\begin{Bmatrix} \sigma^{(k+1)} \\ \kappa^{(k+1)} \end{Bmatrix} = \begin{Bmatrix} \sigma^{(k)} \\ \kappa^{(k)} \end{Bmatrix} - [\mathbf{J}_p(\sigma^{(k)}, \kappa^{(k)})]^{-1} \begin{Bmatrix} R^{(k)} \end{Bmatrix}$$

$$L = \frac{\partial \sigma}{\partial \epsilon}$$

with $\mathbf{J}_p(\sigma^{(k)}, \kappa^{(k)}) = \begin{bmatrix} \dfrac{\partial R_\epsilon}{\partial \sigma} & \dfrac{\partial R_\epsilon}{\partial \kappa} \\ \dfrac{\partial R_f}{\partial \sigma} & \dfrac{\partial R_f}{\partial \kappa} \end{bmatrix}$

Computation of internal forces - summary

$$^{t+\Delta t}\{f_{int}\}^{(i)} = \int_{V_e} [B]\ ^{t+\Delta t}\{\sigma\}^{(i)}\ \mathrm{dV_e}$$

- Determine $^{t+\Delta t}\{\sigma\}^{(i)}$, knowing that the yield surface evolves
- A system of equations needs to be solved
- This system is nonlinear if $\sigma_y(\kappa)$ is a nonlinear function

‘Local’ (at each Gauss point) problem solved by Newton Raphson

- Initialise using the elastic predictor
- if outside of the yield surface ‘bring back’ the point on the yield surface by solving the equations

Material tangent stiffness = by-product of this stress update

$$\{\Delta\sigma\} = \underbrace{^{t}\left[\frac{\partial\sigma}{\partial\varepsilon}\right]}_{^{t}[\mathbf{L}]}\{\Delta\varepsilon\}$$

Loops in the solution procedure for plasticity

Loop on loading (for) 1

Initialise residual

Loop on iterations (while residual > tolerance) 2

Assembly of stiffness (Loop on elements) 3

$$ {}^{t+\Delta t}[K_t]^{(i-1)} = \sum_{e} \left(\int_{V_e} [B]^T \; {}^{t+\Delta t}[\mathbf{L}]^{(i-1)} \, [B] \, \mathrm{dV_e} \right) $$

Computation reaction forces

Compute internal forces (loop on elements) 4

$$ {}^{t+\Delta t}\{f_{int}\}^{(i)} = \int_{V_e} [B] \; {}^{t+\Delta t}\{\sigma\}^{(i)} \; \mathrm{dV_e} $$

Compute stresses at Gauss points

Local Newton Raphson at Gauss points (bring back point on yield) 5

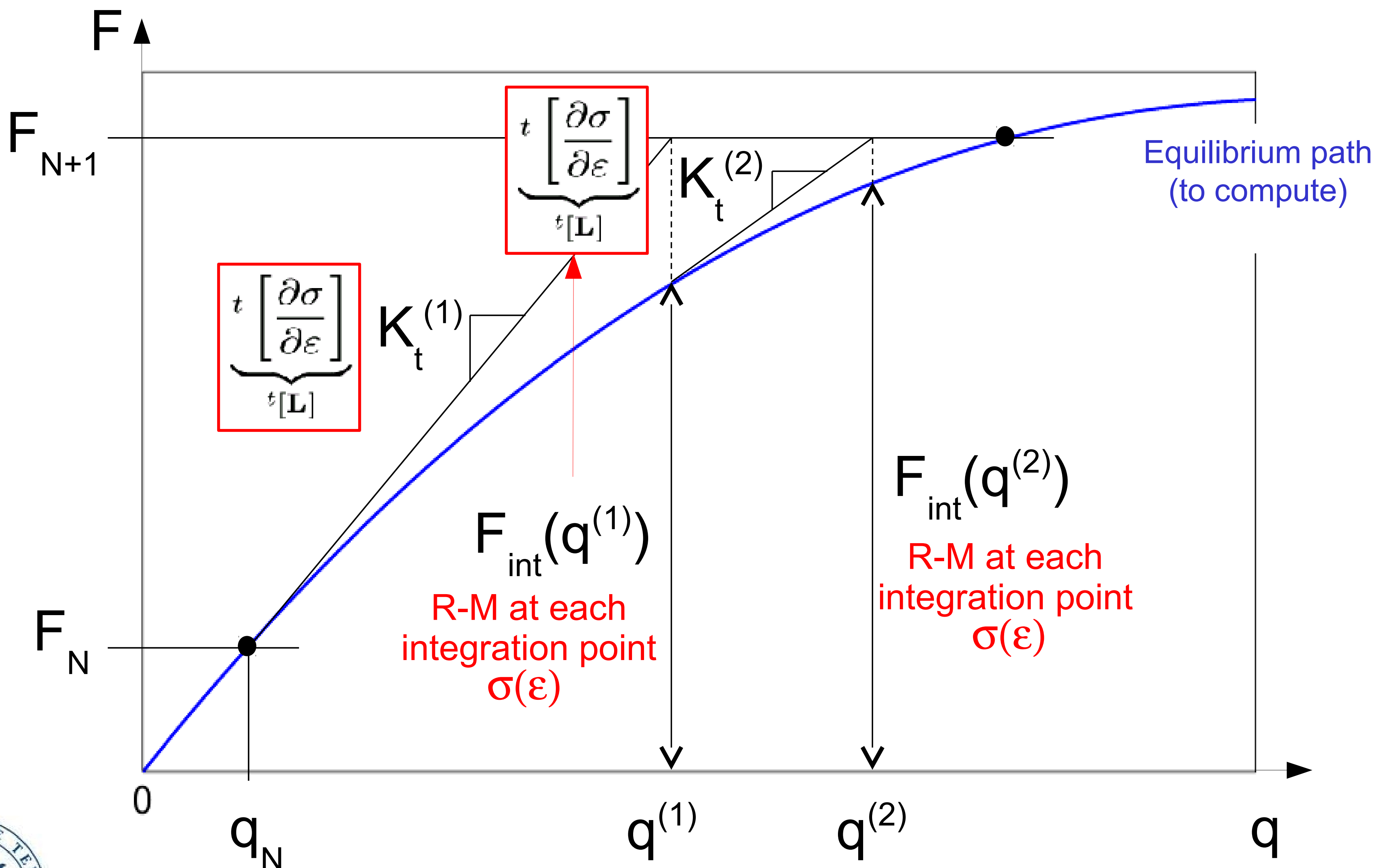
Evaluate new residual

End of iteration loop

End of loop on loading

Structure of a NL FE code

Graphical interpretation



Plasticity - summary

- Permanent strain

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

- Constitutive law (in 1D)

$$\sigma = E\varepsilon^e = E\left(\varepsilon - \varepsilon^p\right)$$

- Stress treshold

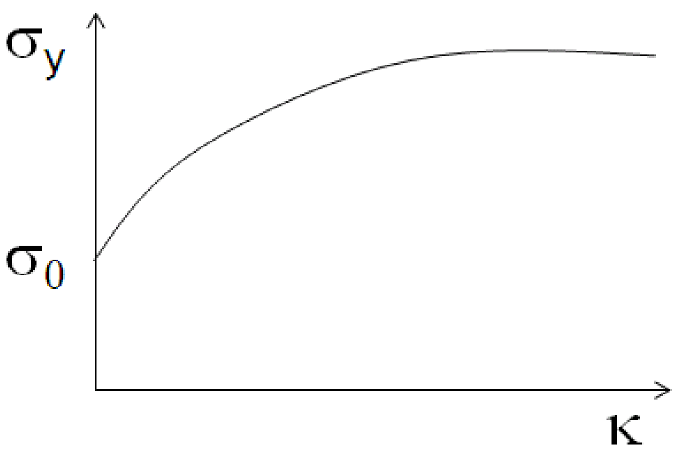
$$f^p = \sigma_{eq}\left(\sigma_1, \sigma_2, \sigma_3\right) - \sigma_y\left(\kappa\right) = 0$$

- History variable

Cumulated plastic strain

- Hardening/evolution law

$$\sigma_y\left(\kappa\right)$$



- Implicit nature; RM at Gauss pts

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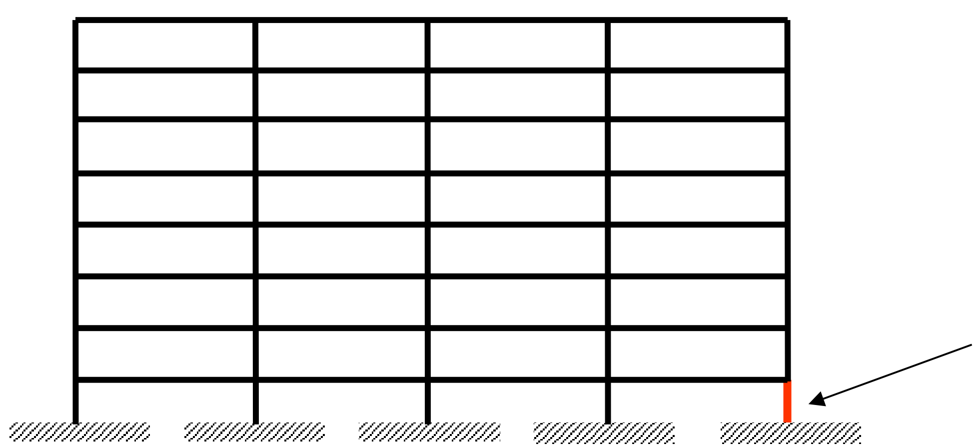
Incremental elastic approach

Suppress a structural element

(Origin of the collapse)

Structural check

Sequence of elastic computations



Initially failing elements

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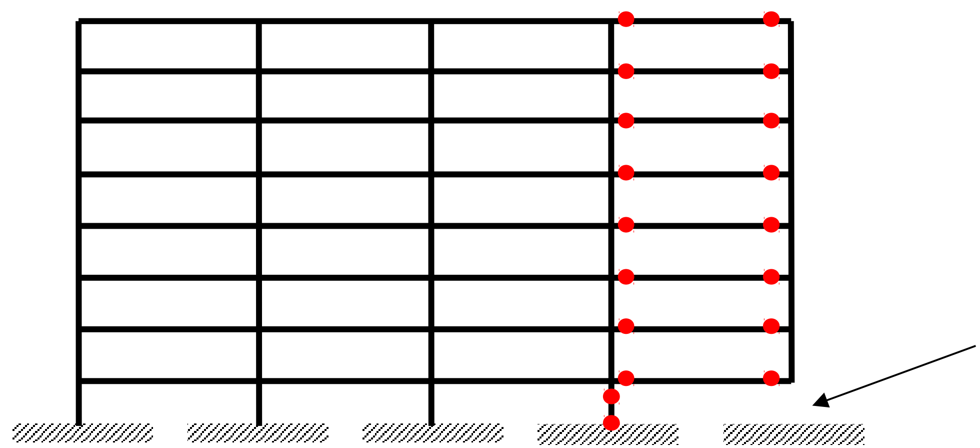
Incremental elastic approach

Suppress a structural element

(Origin of the collapse)

Structural check

Sequence of elastic computations



Initially failing elements

Example: beam-column frame

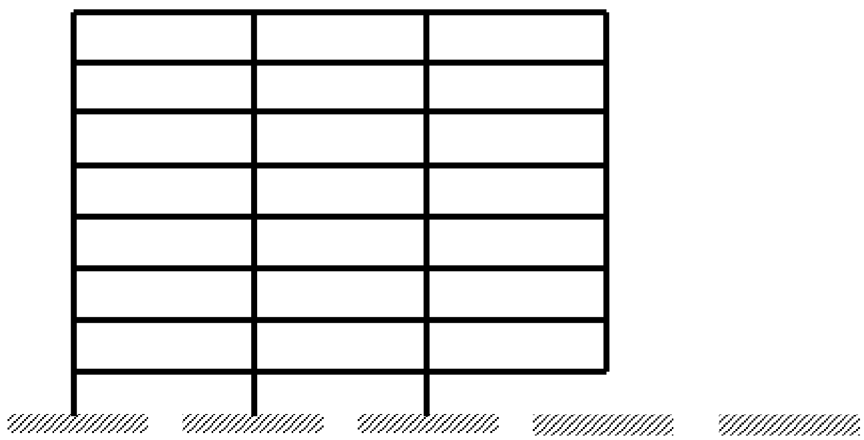
Incremental elastic approach

Suppress a structural element

(Origin of the collapse)

Structural check

Sequence of elastic computations



Suppress the failing elements in the successive elastic steps

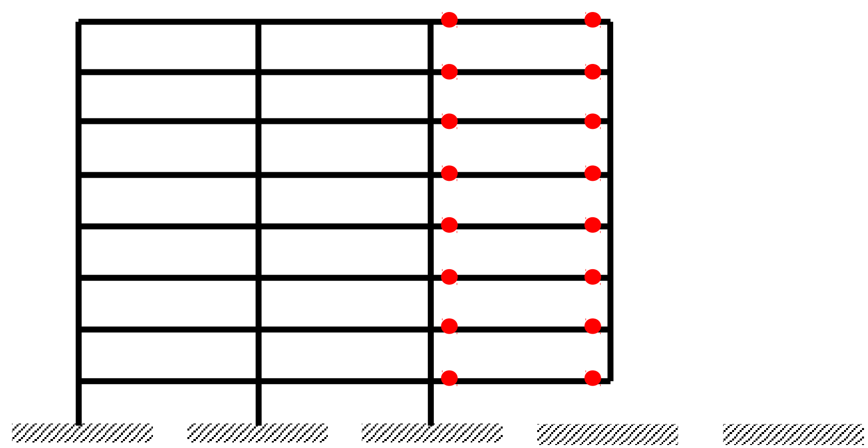
Incremental elastic approach

Suppress a structural element

(Origin of the collapse)

Structural check

Sequence of elastic computations



Suppress the failing elements in the successive elastic steps

Incremental elastic approach

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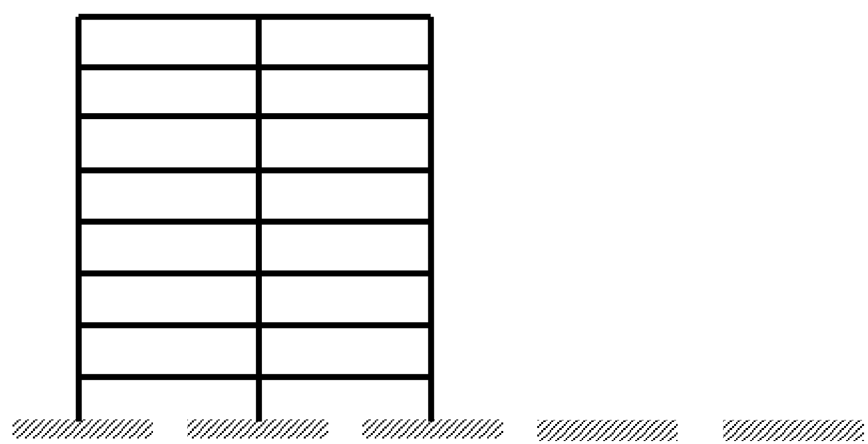
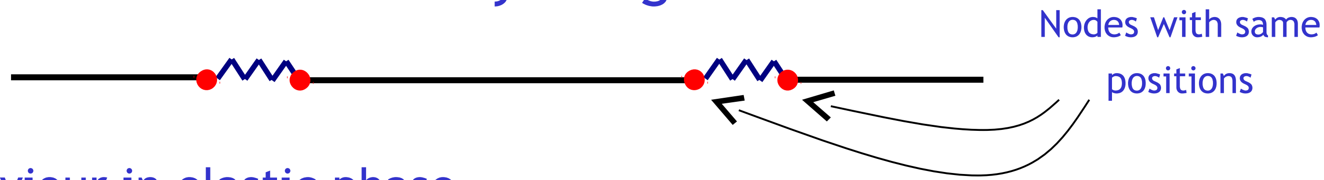
Sequence of elastic computations



Equilibrium is reached: no failing element is found anymore

Plastic behavior of beam-column frames

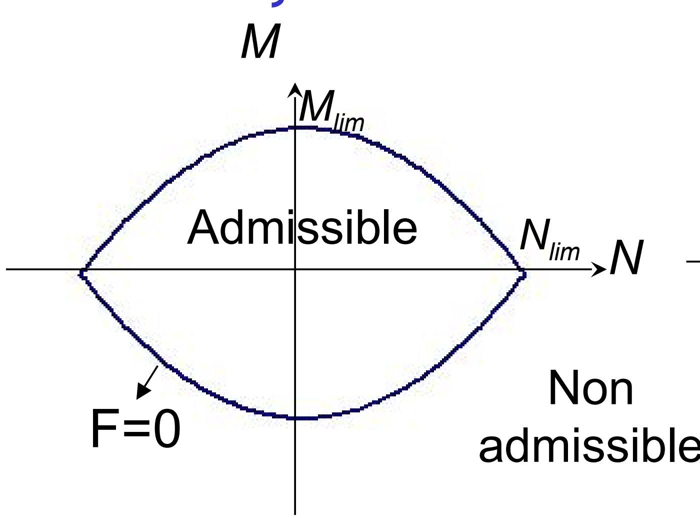
Elastic beam elements linked by ‘hinges’



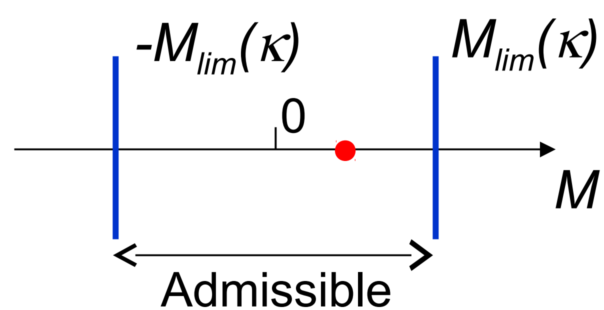
Rigid behaviour in elastic phase

Plastic behaviour after reaching plastic moment limit

Plasticity criteria in M-N variables

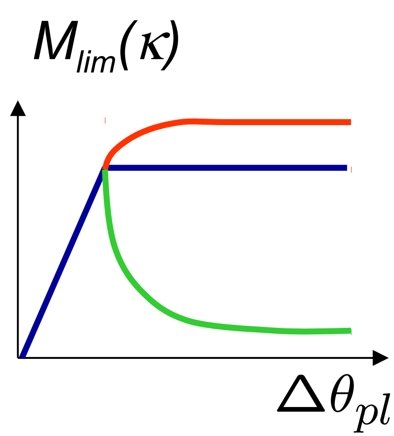


Plastification in (M-N) - Symmetric cross-section with identical behaviour in tension and in compression



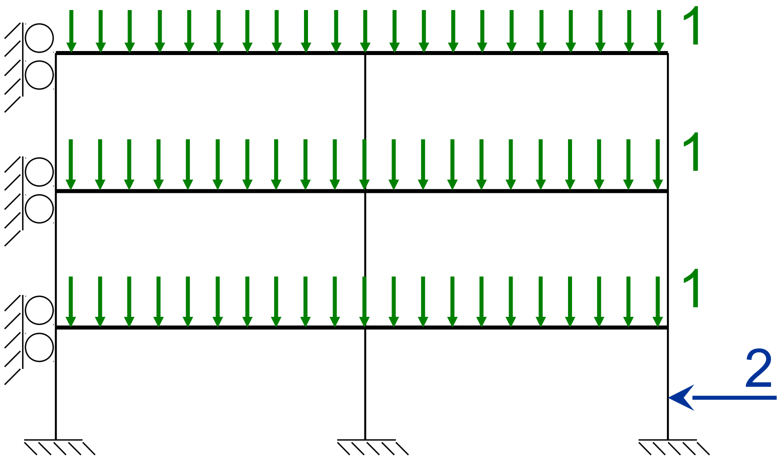
Plastification by M only (simplification)

Evolution law



Comparison with a nonlinear approach

Complete non linear computation on a 'simple' structure



Simulate (in simplified way) the cause of the initial failure

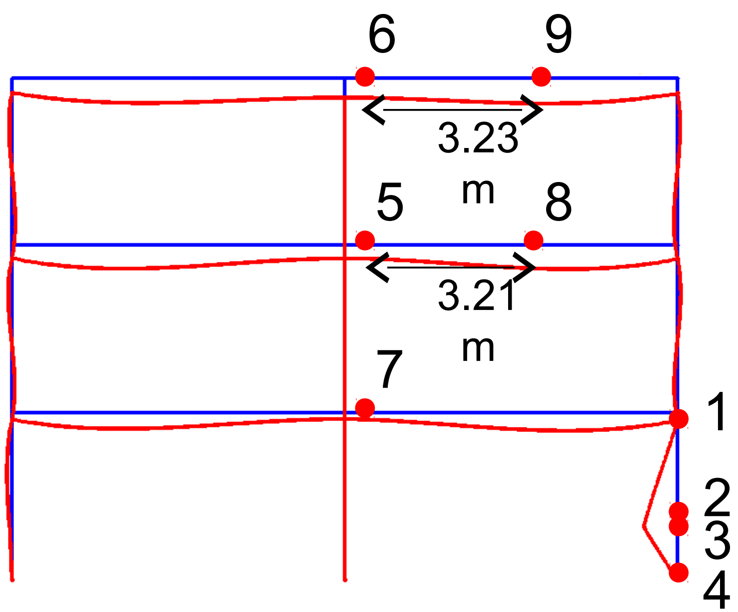
The failing element is present during the whole computation

Large displacements (to model the instability of the initially failed element)

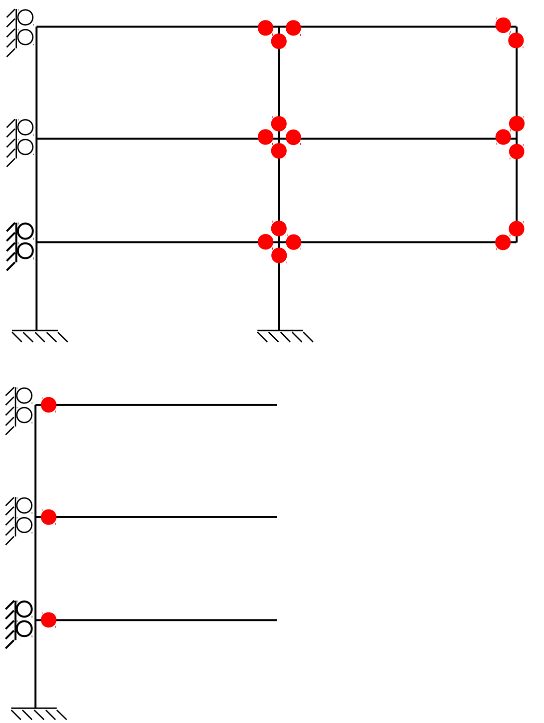
Example: beam-column frame

Comparison with a nonlinear approach

Complete computation



Elastic successive steps



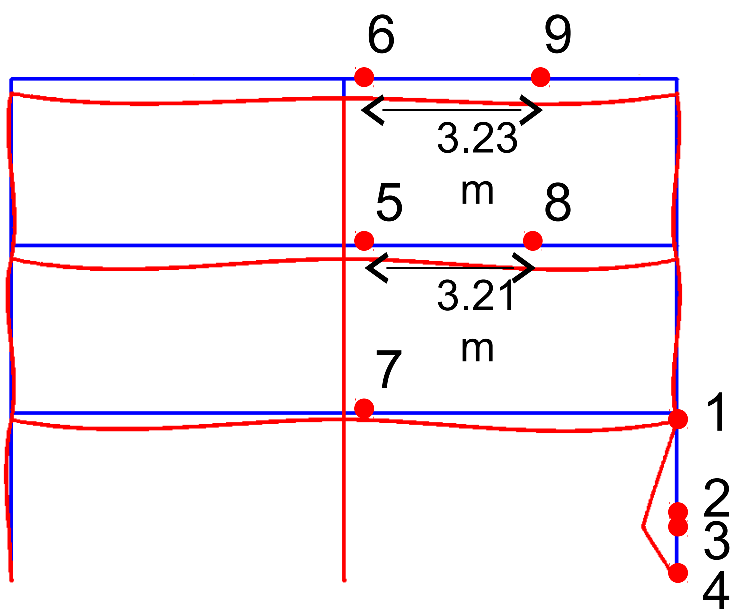
The failure schemes are completely different

Example: beam-column frame

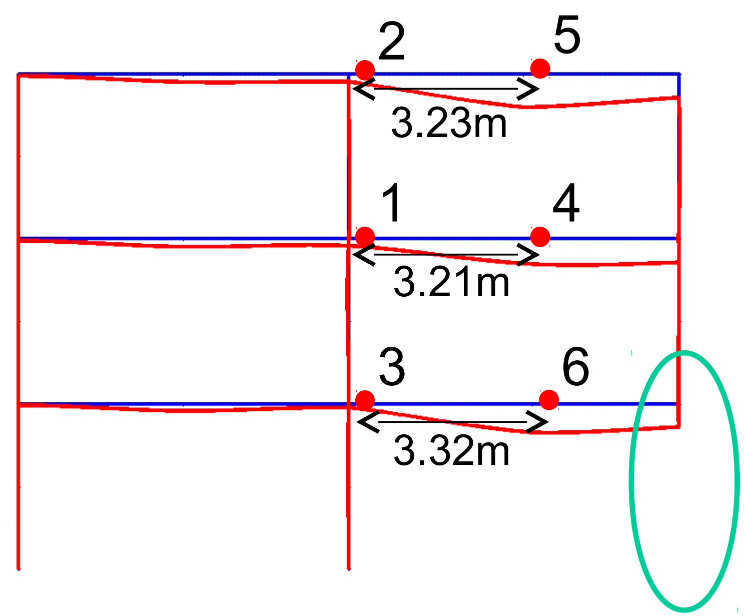
Comparison with a nonlinear approach

Initially failing element replaced by its forces which are decreased progressively

Complete computation



'Incomplete' non linear computation



Replace failing element by its M,N,T
Decrease these M,N,T progressively

The failure modes are much more in agreement

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Recommended literature

M.A. **Crisfield**, Non-linear Finite Element Analysis of Solids and Structures **Volume 1**: ESSENTIALS. John Wiley & Sons Ltd. Bafins Lane, Chichester West Sussex PO19 IUD, England, 1991.

B.S. Iribarren, P. Berke, Ph. Bouillard, J. Vantomme, T.J. Massart, Investigation of the influence of design and material parameters in the progressive collapse analysis of RC structures, *Engineering Structures*, Vol. 33, page 2805-2820, 2011.

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