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PROGRESSIVE COLLAPSE:
A MULTISCALE APPROACH ACCOUNTING FOR LARGE DISPLACEMENTS IN RC STRUCTURES

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INTRODUCTION

- EXAMPLES:

Ronan Point

(England, 1968)

Murrah Office

(USA, 1995)

Real Class

(Brazil, 2011)

INTRODUCTION

- Progressive Collapse: catastrophic dynamic behaviour

INITIAL DAMAGE OF STRUCTURAL KEY ELEMENT

REDISTRIBUTION OF LOADS

MORE DAMAGE

STABILIZATION or COMPLETE FAILURE

Dynamics

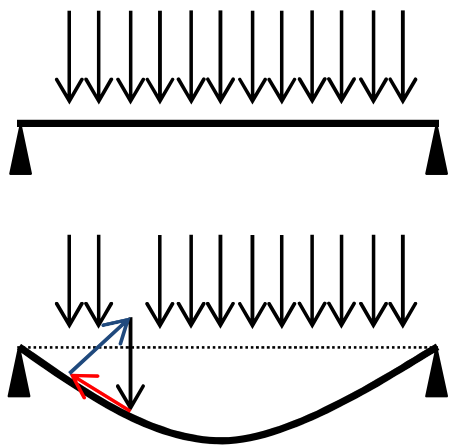
Material Nonlinearities

(Plasticity, Concrete Crushing, strain rates)

Geometrical Nonlinearites

GEOMETRICAL NONLINEAR EFFECTS

- Changes in geometry have significant influence on the load deformation behavior

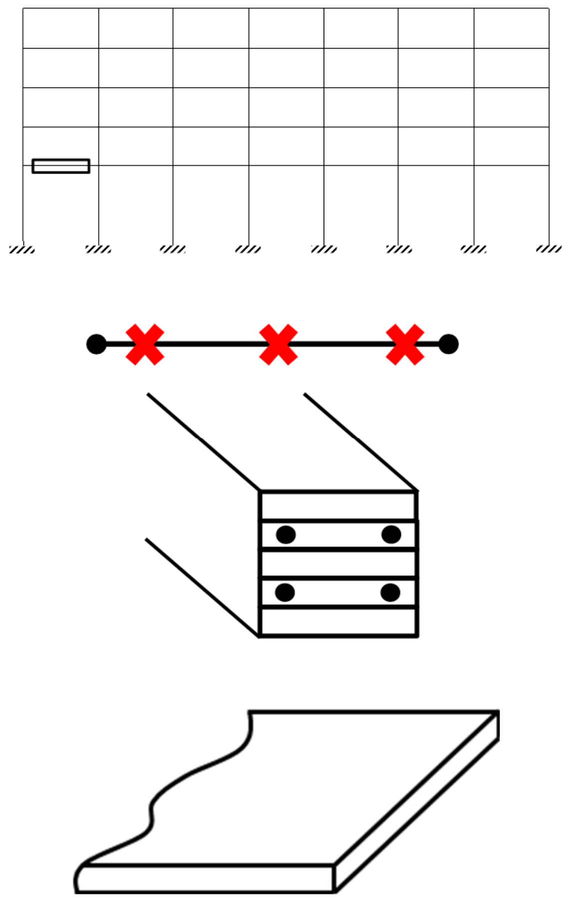


Catenary effect

- catenary effect helps mitigating progressive collapse[1]

(1) Tan, K. H., Pham, X. D., Membrane actions of RC slabs in mitigating progressive collapse of building structures. ***Proceedings of the Design and Analysis of Protective Structures***, Singapore, 2010.

MULTISCALE APPROACH



structural level

finite element level

sectional level at Gauss points

layer level

MAIN CONTRIBUTION

COROTATIONAL BEAM FORMULATION

(GEOMETRICALLY NONLINEAR EFFECTS)[1,2]

+

NONLINEAR DYNAMIC MULTILAYERED BEAM APPROACH[3]

ORGANIZATION

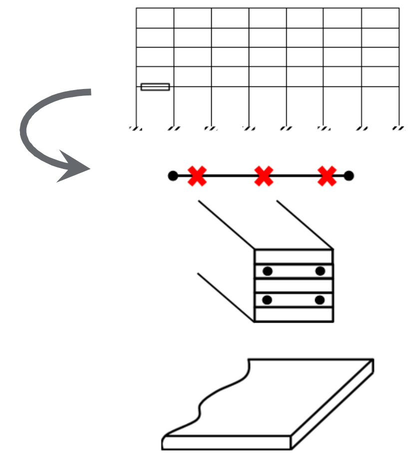
- NONLINEAR FORMULATION – INTERNAL FORCE VECTOR & TANGENT STIFFNESS MATRIX
- COMPUTATIONAL RESULTS - CASE STUDY
- CONCLUSION
- PERSPECTIVES OF FUTURE WORK

(1) **Battini, J-M., Corotational beam elements in instability problems,** Ph.D Thesis, Royal Institute of Tecnology, Department of Mechanics, Stockholm, Sweden, 2002.

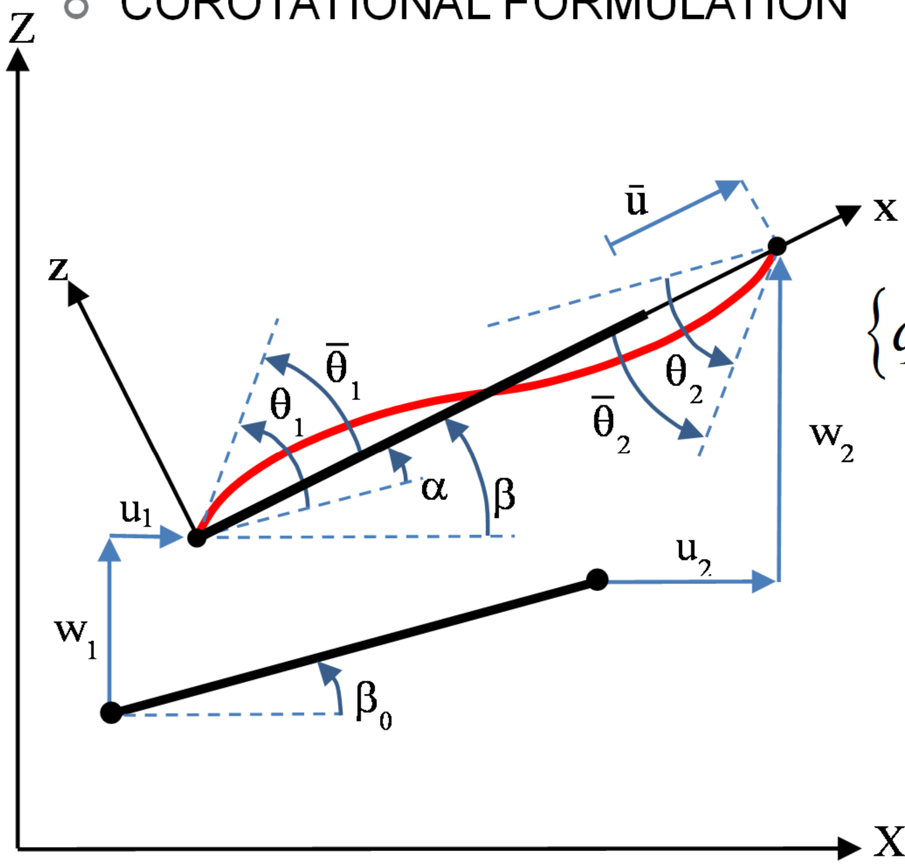
(2) **Crisfield, M. A., Non-Linear Finite Element Analysis of Solids and Structures,** Vol. I. John Wiley & Sons, 1997.

(3) **Iribarren, B. S., Berke, P., Bouillard, Ph., Vantomme, J., Massart, T.J., Investigation of the influence of design and material parameters in the progressive collapse analysis of RC structures,** *Engineering Structures*, Vol. 33, 2805-2820, 2011.

NONLINEAR FORMULATION



- COROTATIONAL FORMULATION



Corotational reference system and kinematic variables

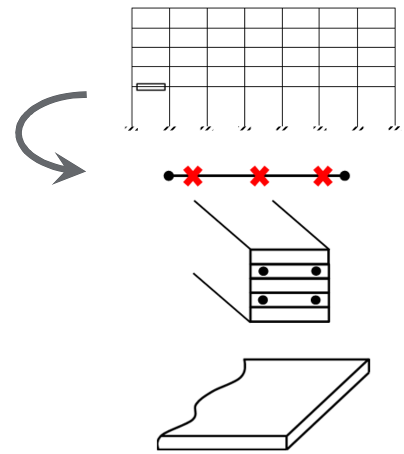
$$\{q_e\}^T = \{u_1 \quad w_1 \quad \theta_1 \quad u_2 \quad w_2 \quad \theta_2\}$$

$$\{\bar{q}_e\}^T = \{\bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2\}$$

$$\begin{bmatrix} \bar{u} \\ \bar{\theta}_1 \\ \bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} l_f - l_i \\ \theta_1 - \alpha \\ \theta_2 - \alpha \end{bmatrix} = \begin{bmatrix} l_f - l_i \\ \theta_1 - \beta - \beta_0 \\ \theta_2 - \beta - \beta_0 \end{bmatrix}$$

Rotations and deformation are decopled!!!

NONLINEAR FORMULATION



- COROTATIONAL FORMULATION

Local displacements:

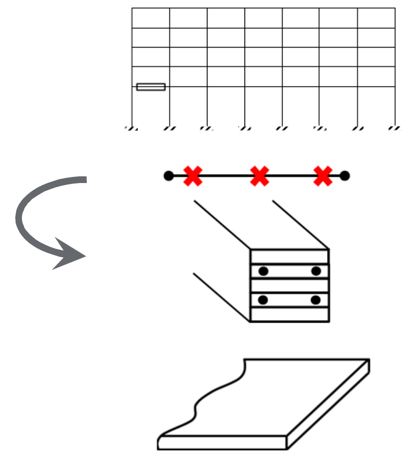
$$\{\bar{q}_e\}^T = \{\bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2\}$$

Interpolated displacements:

$$u = \frac{x}{l_f}\bar{u}$$

$$w = x\left(1 - \frac{x}{l_f}\right)^2 \bar{\theta}_1 + \frac{x^2}{l_f}\left(\frac{x}{l_f} - 1\right)\bar{\theta}_2$$

NONLINEAR FORMULATION

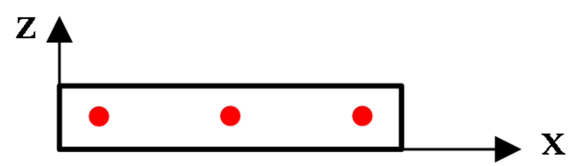


- COROTATIONAL FORMULATION

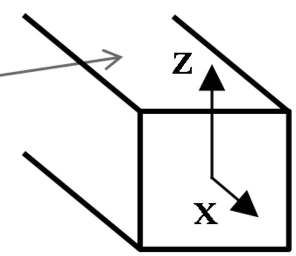
Generalized strains

$$\bar{\varepsilon} = \frac{\partial u}{\partial x} = \frac{\bar{u}}{l_f}$$

$$\chi = \frac{\partial^2 w}{\partial x^2} = \left(-\frac{4}{l_f} + \frac{6}{l_f^{\,2}} x\right)\bar{\theta}_1 + \left(-\frac{2}{l_f} + \frac{6}{l_f^{\,2}} x\right)\bar{\theta}_2$$



Total axial strain at different cross sectional heights

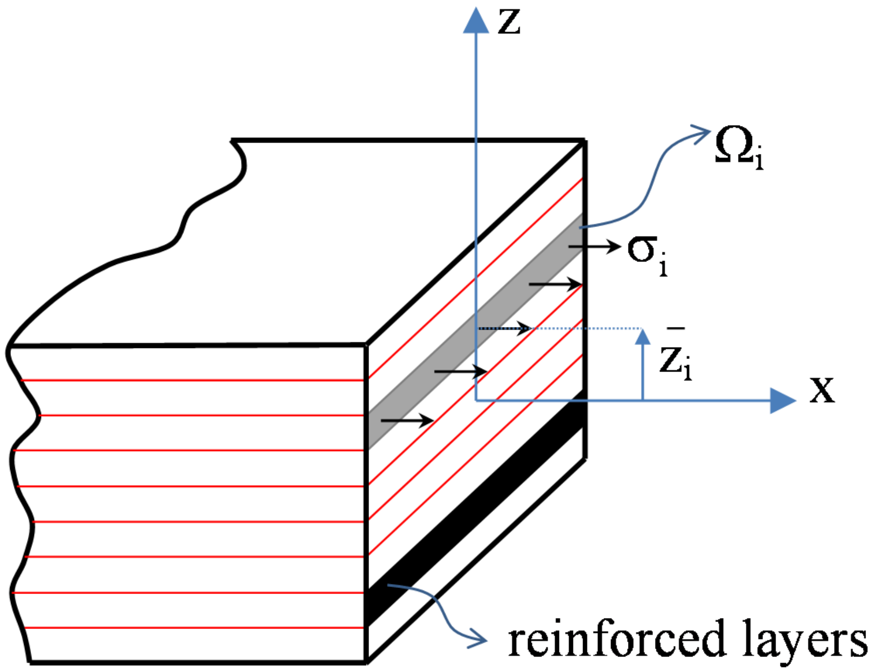


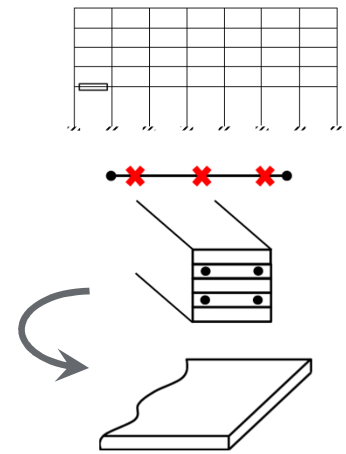
$$\varepsilon = \bar{\varepsilon} - z\chi$$

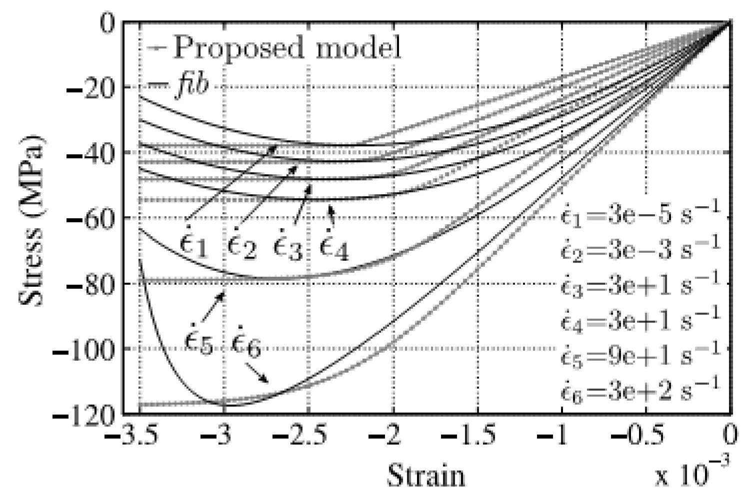
$$\sigma(\varepsilon) => \sigma(x, z)$$

NONLINEAR FORMULATION

- MULTILAYERED APPROACH[1]

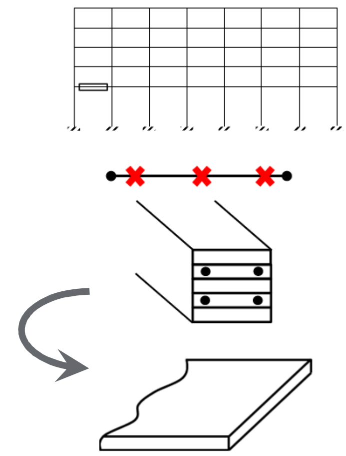




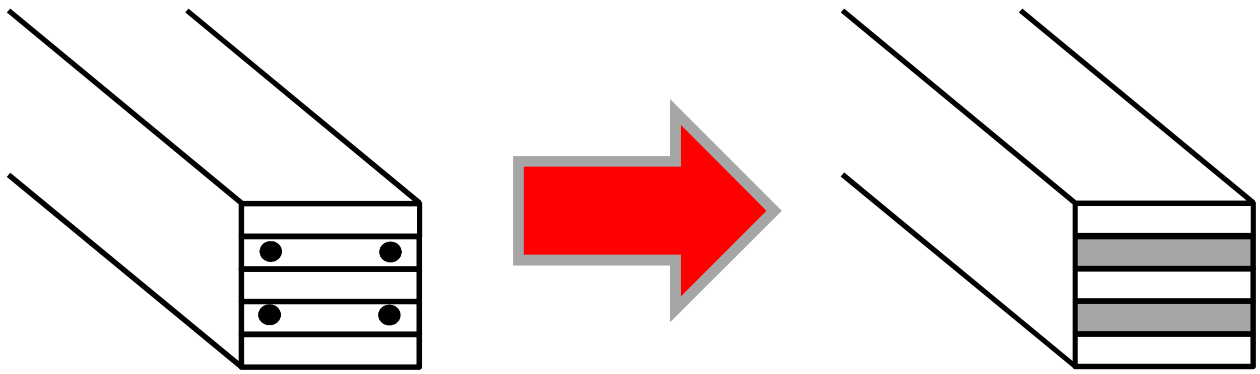


$$\sigma_{concrete}\left(\varepsilon,\dot{\varepsilon}\right)$$

NONLINEAR FORMULATION



- MULTILAYERED APPROACH[1]



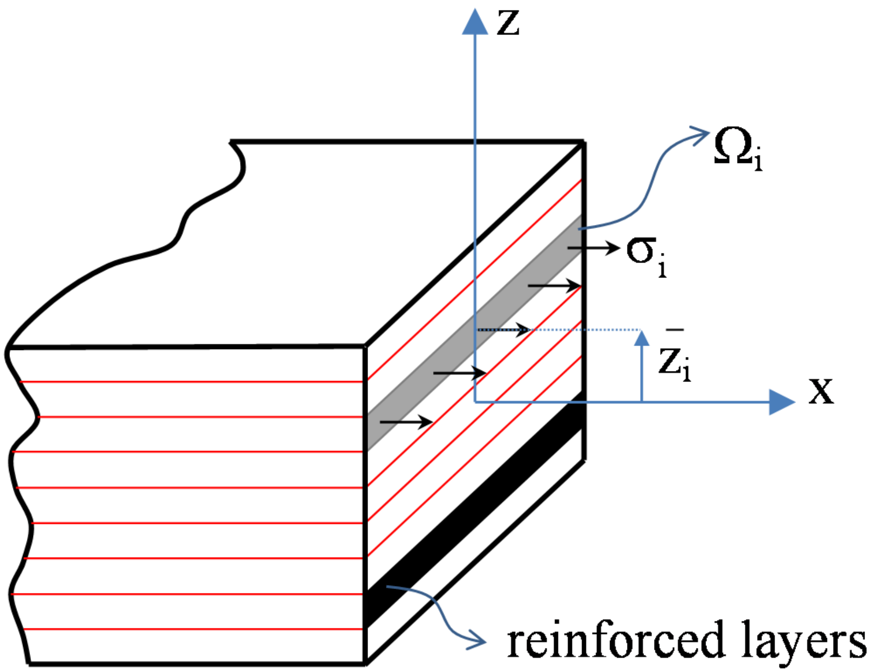
- "MIXTURE" TOTAL STRESSES

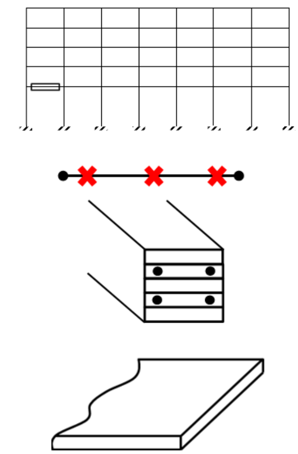
$$\sigma_{i,TOTAL} = \%concrete\ x\ \sigma_{i,concrete} + \%steel\ x\ \sigma_{i,steel}$$

(1) Iribarren, B. S., Berke, P., Bouillard, Ph., Vantomme, J., Massart, T.J., Investigation of the influence of design and material parameters in the progressive collapse analysis of RC structures, ***Engineering Structures***, Vol. **33**, 2805-2820, 2011.

NONLINEAR FORMULATION

- MULTILAYERED APPROACH





$$N = \Sigma \sigma_i \Omega_i$$

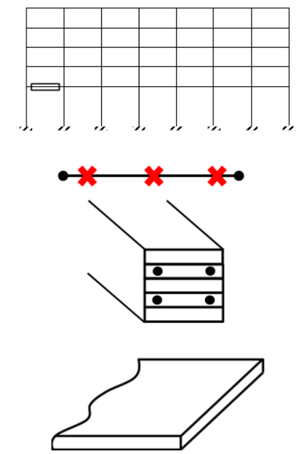
$$M = -\Sigma z_i \sigma_i \Omega_i$$

$$\bar{N} = \int_{l_f} \frac{N}{l_f} dx$$

$$\bar{M}_1 = \int_{l_f} M \left(6\frac{x}{l_f^{\,2}} - \frac{4}{l_f} \right) dx$$

$$\bar{M}_2 = \int_{l_f} M \left(6\frac{x}{l_f^{\,2}} - \frac{2}{l_f} \right) dx$$

NONLINEAR FORMULATION



- TRANSFORMATION MATRIX

$$\begin{bmatrix} \delta\bar{u} \\ \delta\bar{\theta}_1 \\ \delta\bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1/l_i & 1/l_i \end{bmatrix} \begin{bmatrix} -c & -s & 0 & c & s & 0 \\ s & -c & 1 & s & c & 0 \\ s & -c & 0 & s & c & 1 \end{bmatrix} \delta\{q_G\}$$

$$\delta\mathbf{q}_L \quad = \mathbf{T}\,\delta\mathbf{q}_G$$

$$\begin{aligned} \mathbf{f}_{\text{int},L}(GEOM.) &= \left\{ f_1^x \quad f_1^y \quad c_1 \quad f_2^x \quad f_2^y \quad c_2 \right\}^t \\ &= \mathbf{T}^{-1}\left\{ \bar{N} \quad \bar{M}_1 \quad \bar{M}_2 \right\}^t \end{aligned}$$

NONLINEAR FORMULATION

- TANGENT STIFFNESS MATRIX

$$\mathbf{K}_G = \mathbf{T}^t \mathbf{K}_L \mathbf{T} + N \frac{\mathbf{z}\mathbf{z}^t}{l_f} + \left(M_1 + M_2\right) \frac{1}{l_f^2} \left(\mathbf{r}\mathbf{z}^t + \mathbf{z}\mathbf{r}^t\right)$$

$$\mathbf{K}_L = \mathbf{B}^t \mathbf{H} \mathbf{B}$$

$$\mathbf{r} = \begin{bmatrix} -c & -s & 0 & c & s & 0 \end{bmatrix}^t$$

$$\mathbf{z} = \begin{bmatrix} s & -c & 0 & -s & c & 0 \end{bmatrix}^t$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \left(6\frac{x}{L^2} - \frac{4}{L}\right) & \left(6\frac{x}{L^2} - \frac{2}{L}\right) \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} \Sigma H_i \Omega_i & -\Sigma H_i z_i \Omega_i \\ -\Sigma H_i z_i \Omega_i & -\Sigma H_i z_i^2 \Omega_i \end{bmatrix}$$

$$H_i = \frac{\partial \sigma_i}{\partial \varepsilon_i}$$

CONSISTENT TANGENT OPERATORS

NONLINEAR RESULTS

- CASE STUDY: QUASI-STATIC ANALYSIS OF A COLUMN BEAM ASSEMBLAGE[1,2]

- Conducted by NIST; Detailed on Sadek et al. (2011) and Lew (2011)
- Ten-storey RC framed building – 3 columns, 2 beams
- Designed according to ACI318-02[3]

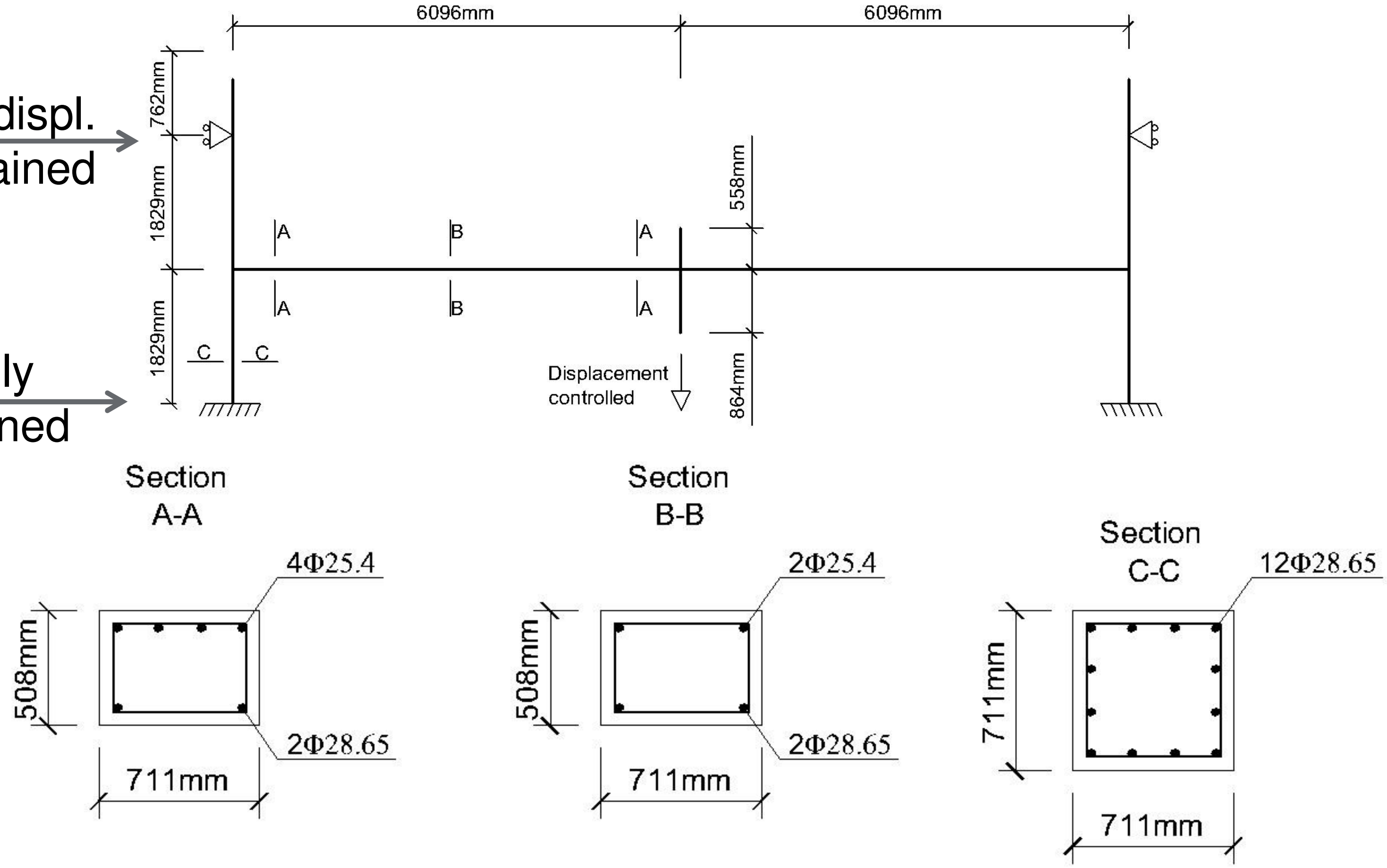
(1) Sadek, F., Main, J. A., Lew, H. S., and Bao, Y., Testing and Analysis of Steel and Concrete Beam-Column Assemblies under a Column Removal Scenario. ***Journal of Structural Engineering***, ASCE **137:9**, 881-892, 2011.
(2) Lew, H. S., Bao, Y., Sadek, F., Main, J. A, Pujol, S. and Sozen, M. A., **An Experimental and Computational Study of Reinforced concrete Assemblies under a Column Removal Scenario**. NIST Technical Note 1720. National Institute of Standards and Technology. U.S.Department of Commerce, 2011.
(3) ACI 318-08. (2008). *Building Code Requirements for Structural Concrete (ACI 318M-08)*. American Concrete Institute, 2007.

NONLINEAR RESULTS

- CASE STUDY: COLUMN BEAM ASSEMBLAGE[1,2]

Hor. displ. restrained

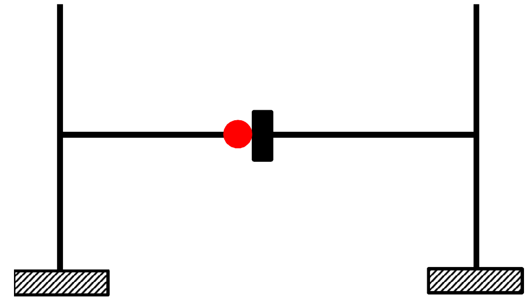
Totally restrained

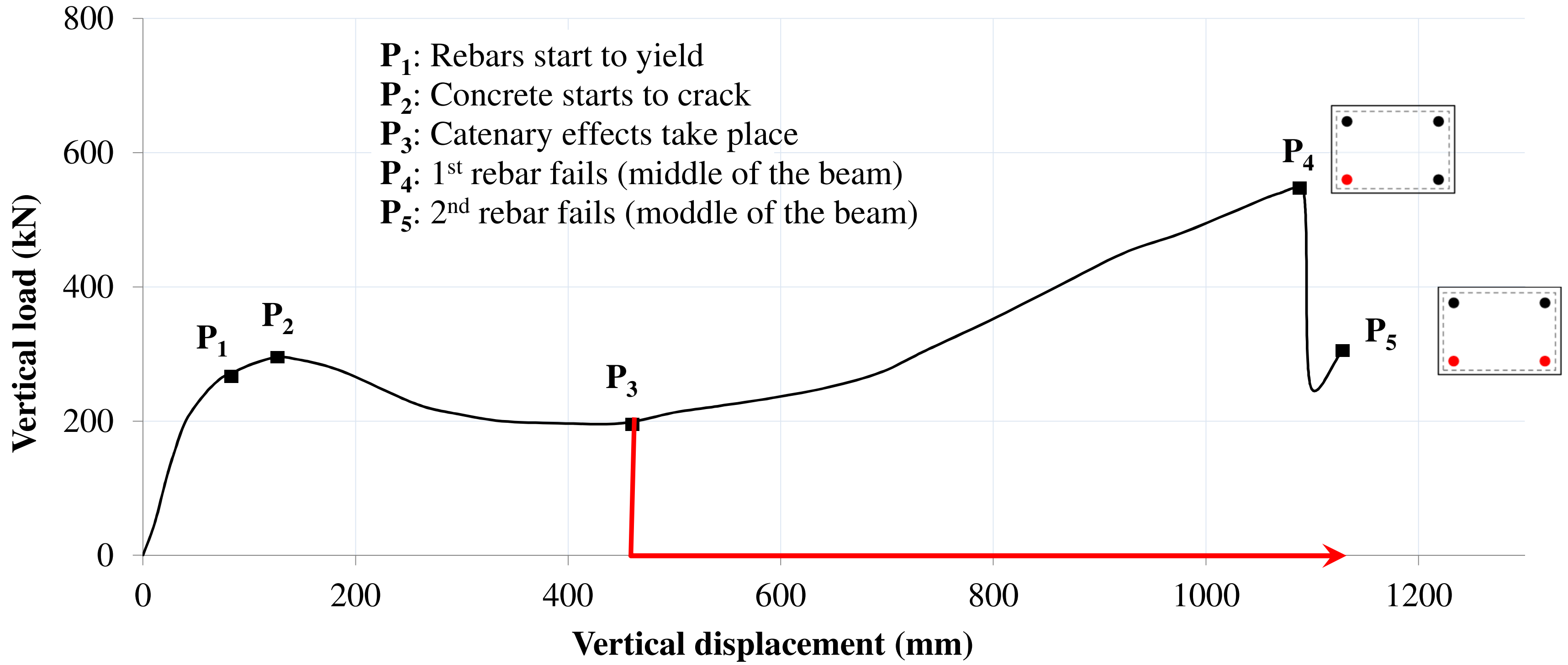


NONLINEAR RESULTS

- CASE STUDY: COLUMN BEAM ASSEMBLAGE[1,2]

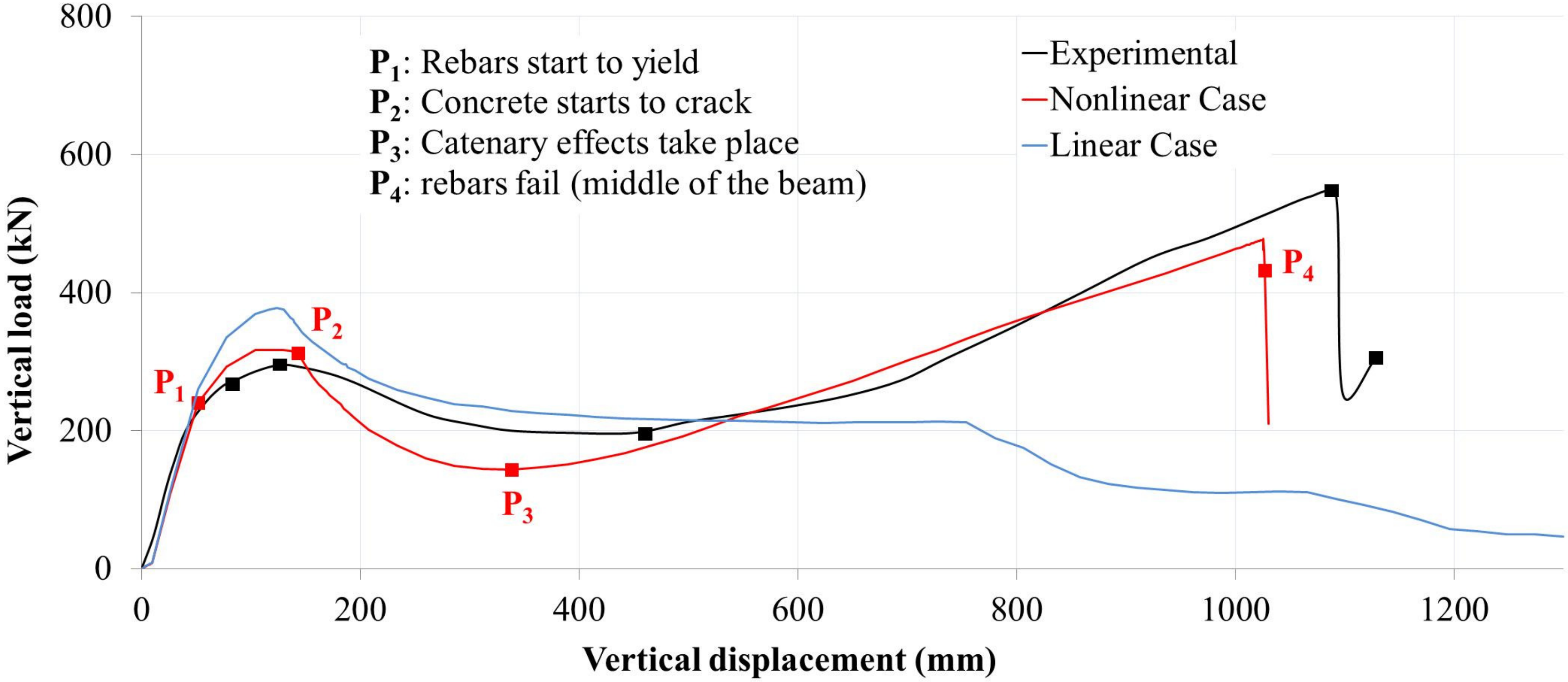
Displacement applied on the middle column after self-weight stabilization

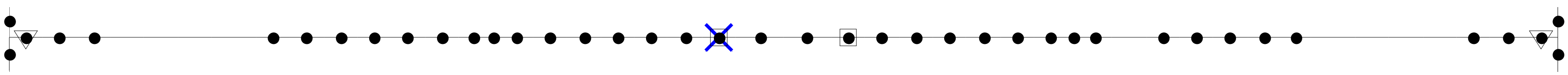




NONLINEAR COMPUTATIONAL RESULTS

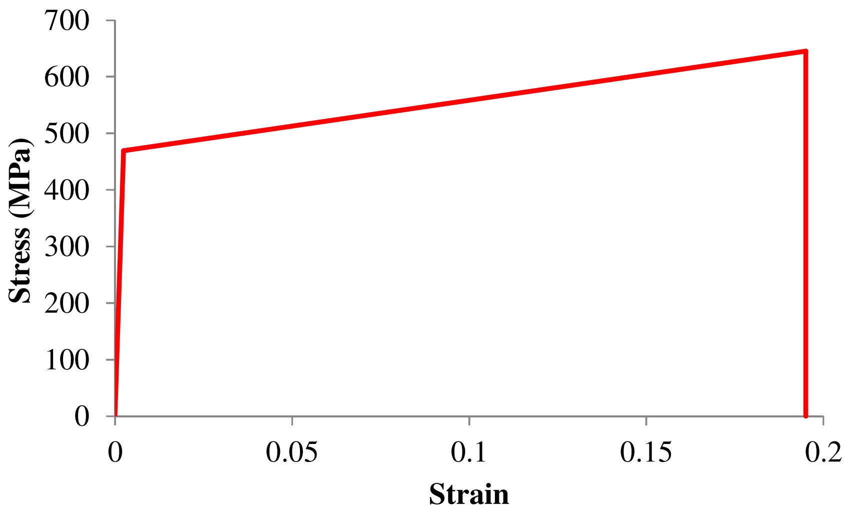
- NUMERICAL RESULTS

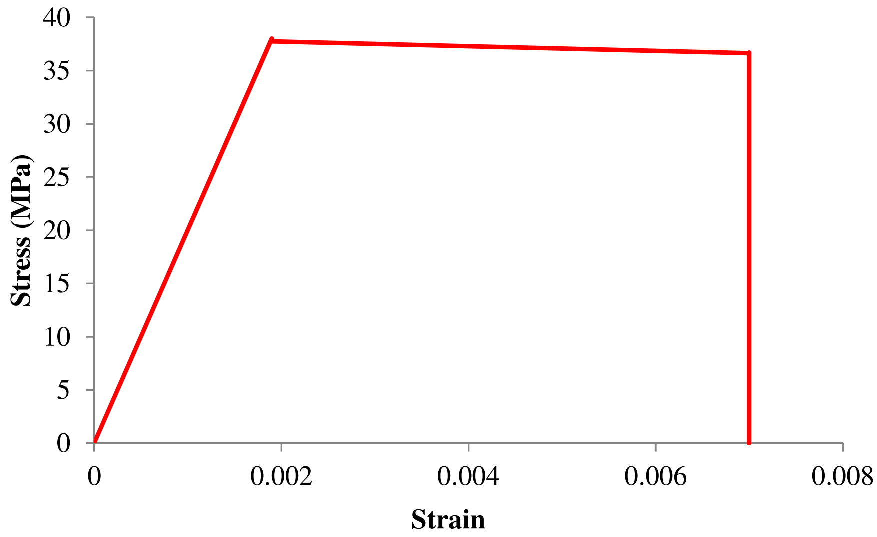




NONLINEAR COMPUTATIONAL RESULTS

- VARIATION OF DATA PARAMETERS





Parameter	
E_{st} (GPa)	200
σ_{st} (MPa)	469
K_{st}	1,88
ε_{st} (%)	19.1

Parameter	Min.
E_{conc} (GPa)	27
σ_{conc} (MPa)	38
K_{conc}	-5
$\varepsilon_{conc.}$ (%)	0.6

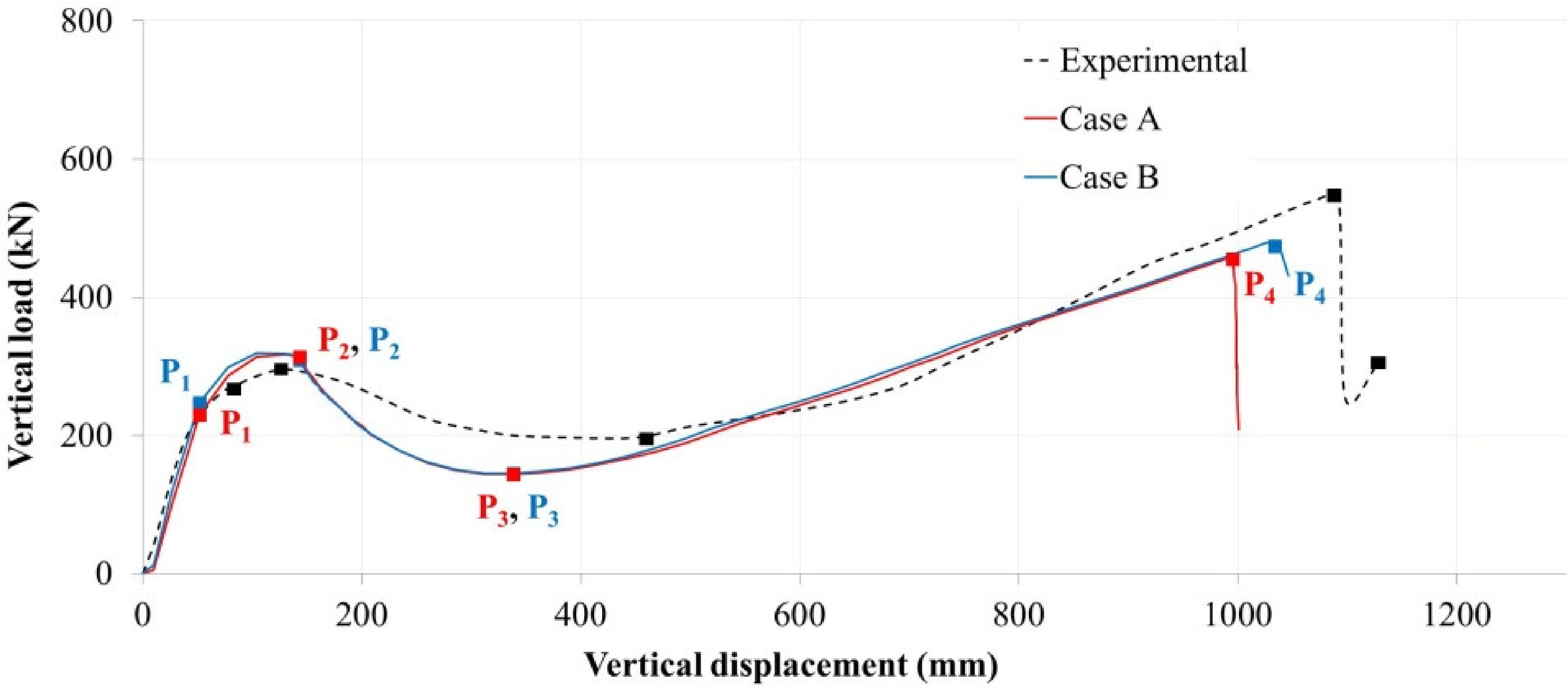
NONLINEAR COMPUTATIONAL RESULTS

- VARIATION OF DATA PARAMETERS

Parameter	-10%	+10%
E_{st} (GPa)	180	220
E_{conc} (GPa)	24	30
σ_{st} (MPa)	420	520
σ_{conc} (MPa)	34	42
K_{st}	0	10
K_{conc}	-10	0
ε_{st}	17	21
ε_{conc}	0,59	0,72

NONLINEAR COMPUTATIONAL RESULTS

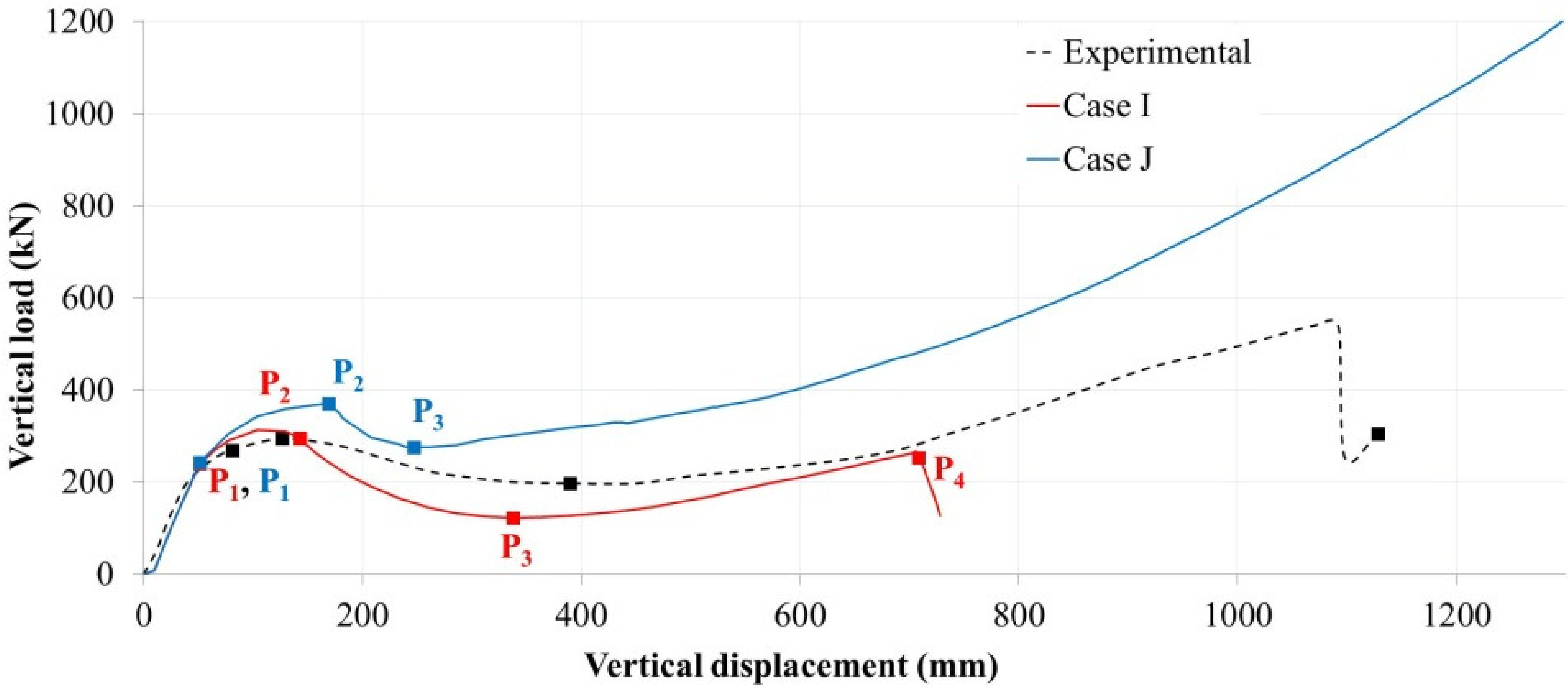
- VARIATION OF DATA PARAMETERS



$180 < E_{st} < 220$ GPa

NONLINEAR COMPUTATIONAL RESULTS

- VARIATION OF DATA PARAMETERS

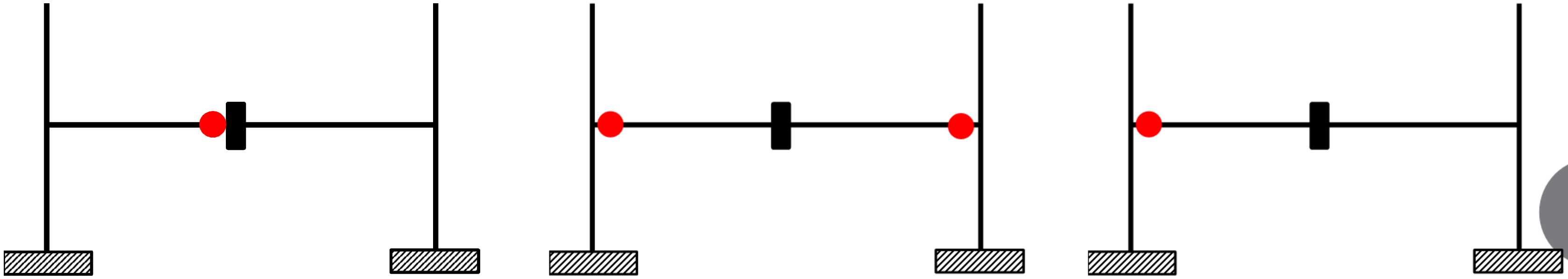


$0 < K_{st} < 10$

NONLINEAR COMPUTATIONAL RESULTS

- VARIATION OF DATA PARAMETERS

Parameter	Min.	Mech.	Max.	Mech.
E_{st} (GPa)	180	Ok	220	nOk
E_{conc} (GPa)	24	nOk	30	Ok
σ_{st} (MPa)	420	Ok	520	nOk
σ_{conc} (MPa)	34	nOk	42	Ok
K_{st}	0	Ok	10	nOk
K_{conc}	-10	Ok	0	Ok
ε_{st}	17	Ok	21	nOk
ε_{conc}	0,59	nOk	0,72	Ok

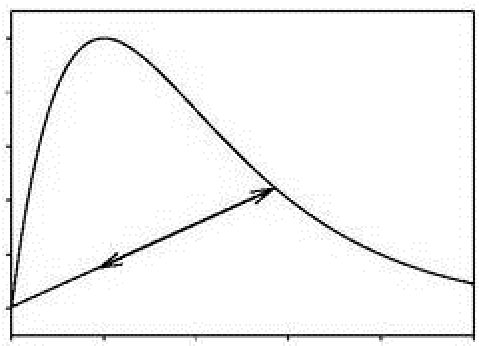


CONCLUSION

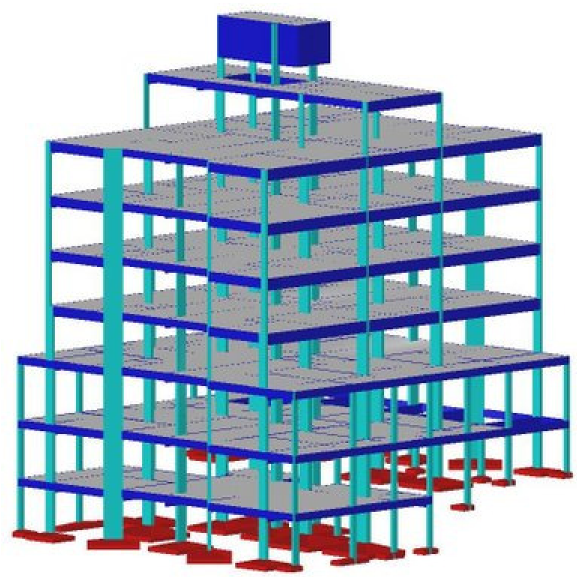
- COROTATIONAL FORMULATION + MULTILAYER FORMULATION
- LARGE DISPLACEMENT BEHAVIOR WAS SUCCESSFULLY DESCRIBED
- AGREEMENT BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS
- ABLE TO REPRODUCE CATENARY EFFECTS
- LOAD DISPLACEMENT CURVE IS NOT SEVERELY AFFECTED BY THE CHANGE OF THE PARAMETERS WHILE THE TYPE OF FAILURE MECHANISM IS VERY SENSITIVE WITHIN THE STUDIED RANGE.

PERSPECTIVES

- UPDATING OF MASS MATRIX AS FUNCTION OF THE DEFORMATION
- INCLUSION OF SHEAR EFFECTS VIA THE TIMOSHENKO BEAM THEORY
- LOADING *VS*. UNLOADING: ASSOCIATION OF PLASTICITY AND DAMAGE



- EXTENSION TO 3D
- PARALLEL COMPUTING



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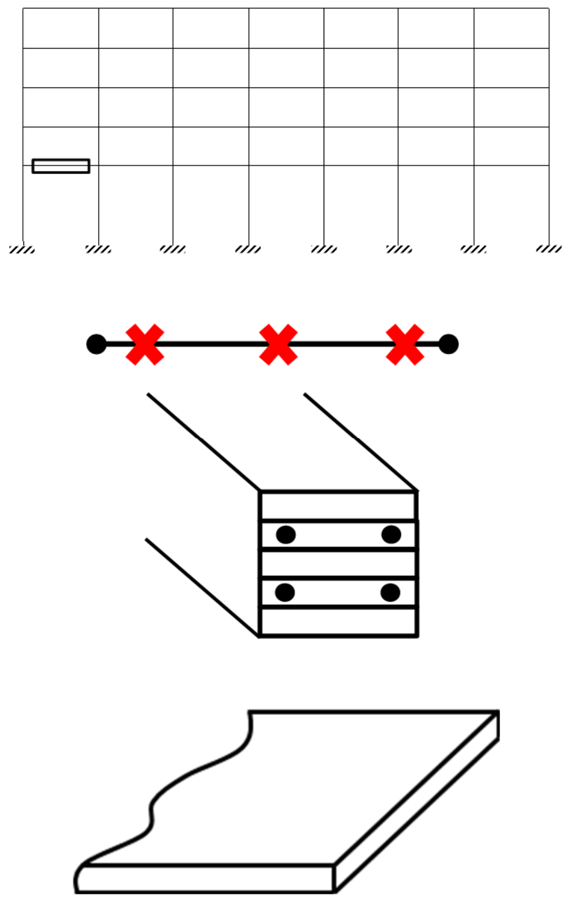
NONLINEAR FORMULATION

- MULTISCALE COMPUTATION OF SECTIONAL STRESSES: PREVIEW



structural level

finite element level

sectional level at
Gauss points

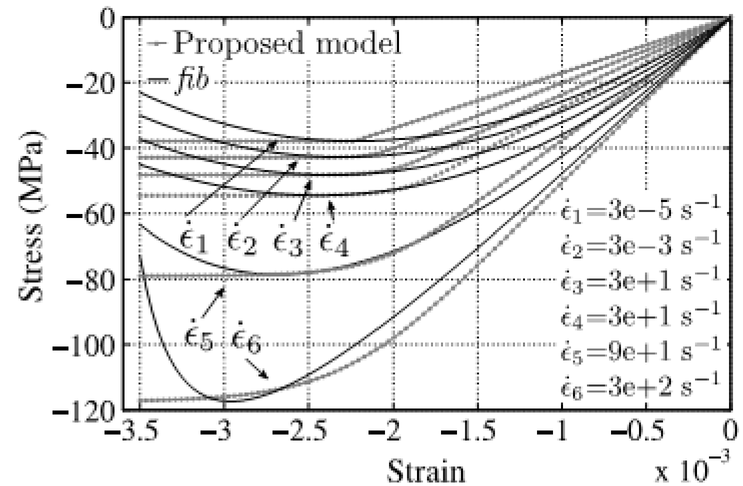
layer level

STRUCTURAL DYNAMIC GOVERNING EQUATIONS[1]

$$\mathbf{f}_{G,\mathrm{int}}(\mathbf{q}_G, \dot{\mathbf{q}}_G) + \mathbf{M}\ddot{\mathbf{q}}_G = \mathbf{f}_{G,ext}$$

$$\mathbf{f}_{G,\mathrm{int}}(\mathbf{q}_G, \dot{\mathbf{q}}_G) = f\left(\mathbf{E}^{gen}, \dot{\mathbf{E}}^{gen}\right)$$

$$\mathbf{J} = \frac{\partial\left[\mathbf{f}_{G,\mathrm{int}}(\mathbf{q}_G, \dot{\mathbf{q}}_G)\right]}{\partial \mathbf{q}_G} + \mathbf{M}\frac{\partial \ddot{\mathbf{q}}_G}{\partial \mathbf{q}_G}$$



(1) Iribarren, B. S., Berke, P., Bouillard, Ph., Vantomme, J., Massart, T.J., Investigation of the influence of design and material parameters in the progressive collapse analysis of RC structures, ***Engineering Structures***, Vol. **33**, 2805-2820, 2011.

NONLINEAR DYNAMIC RESULTS

- CASE STUDY: COLUMN BEAM ASSEMBLAGE[1,2]

Element Type	Rebar Diameter (mm)	Yield Strength (MPa)	Ultimate strength (MPa)	Ultimate Strain (%)
Beam	25.4	476	648	21
Beam	28.65	462	641	18
Column	28.65	483	690	17