UNIVERSIDADE FEDERAL DO CEARÁ

UFOP, Departamento de Engenharia Civil
7th May 2014

PROPEC PROGRAMA DE PÓS GRADUAÇÃO EM ENGENHARIA CIVIL

Nonlinear Multi-Scale Finite Element Modeling of the Progressive Collapse of Reinforced Concrete Structures

Lecture 5: Corotational Bernoulli beam

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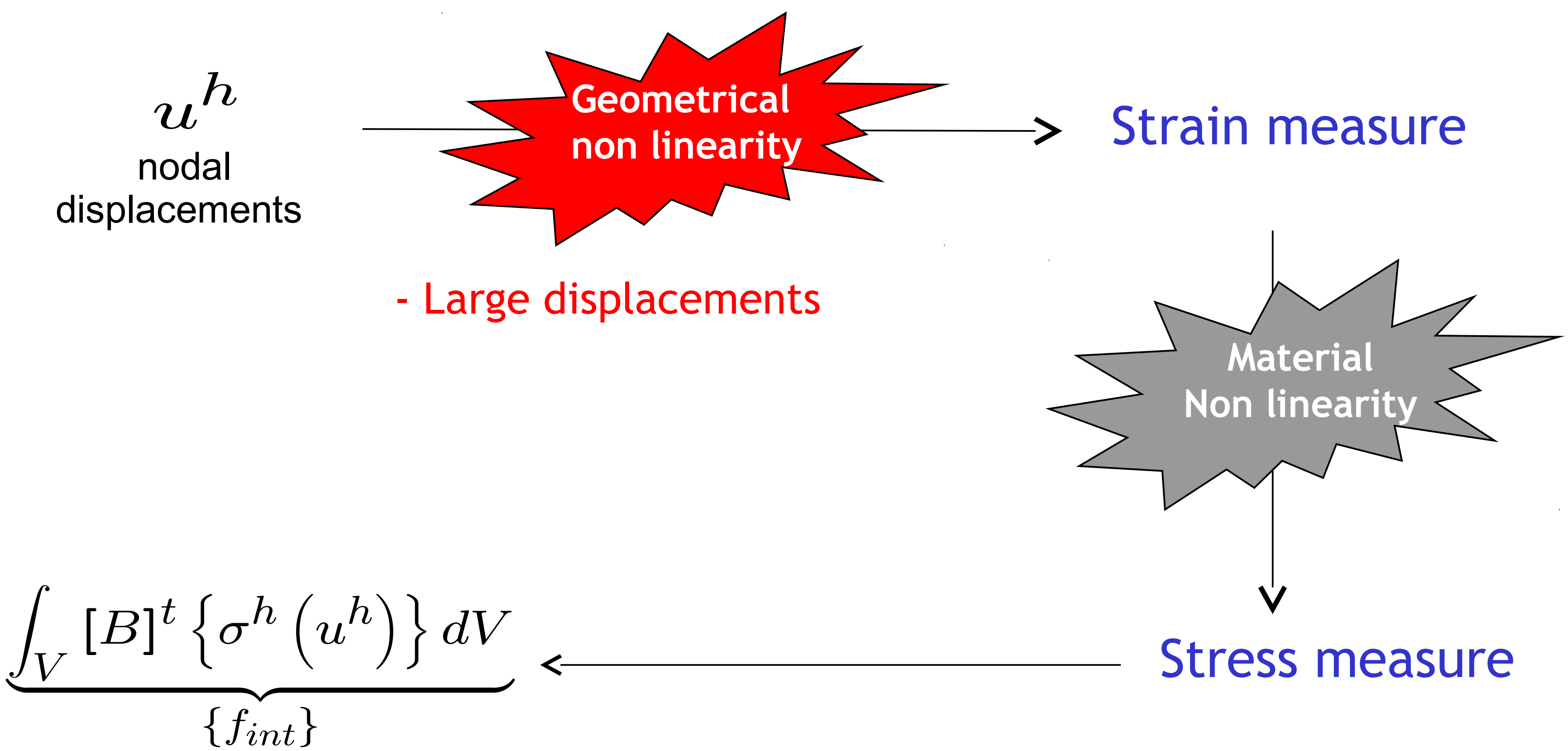
Sources of nonlinearity

Non proportionnality between applied forces and displacements



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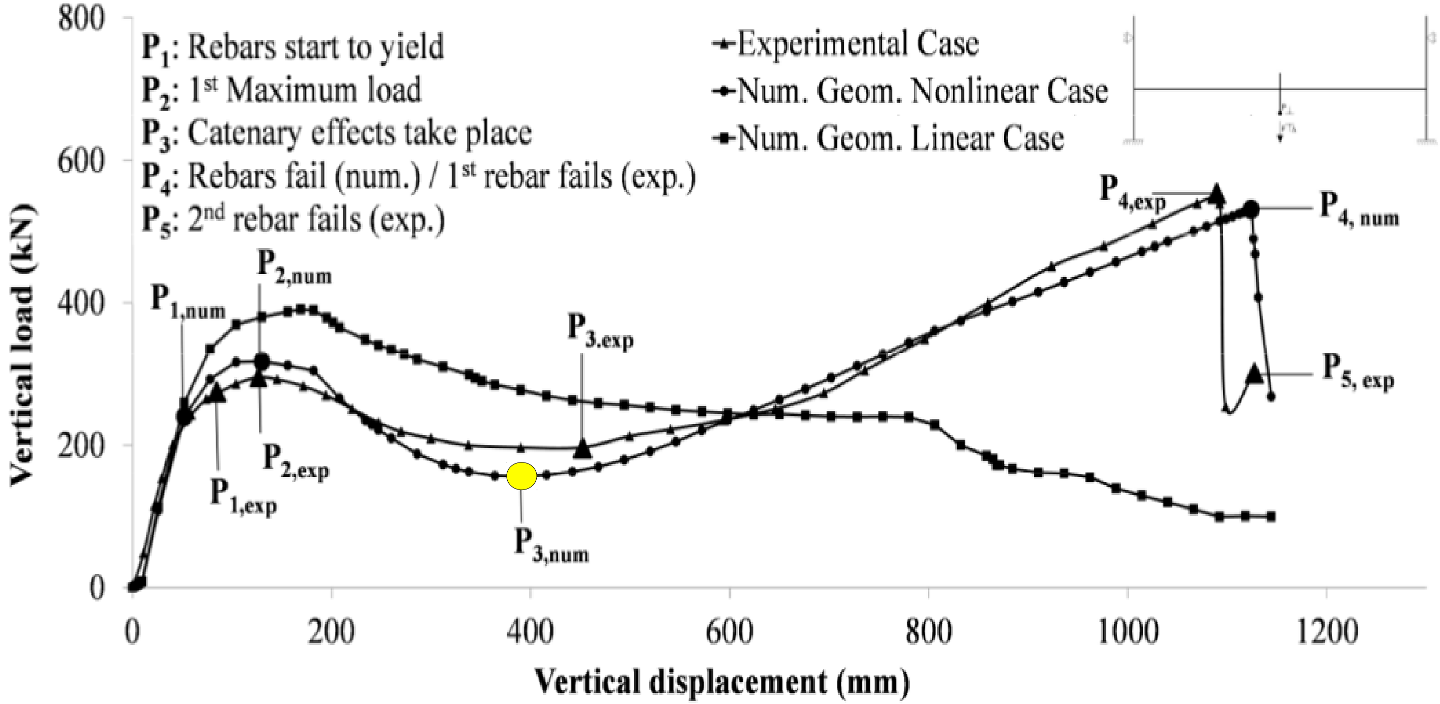
Geometrically NL formulation in PC?

- Possible strength enhancement
- Changes in the shape have to be taken into account
- Finite displacement formulation

Example of nonlinear structural response

[3] F. Sadek et al., *J. Struct. Engng. ASCE*, **137**, 881-892 (2011)

[4] H.S. Lew et al., NIST Technical Note 1720 (2011)



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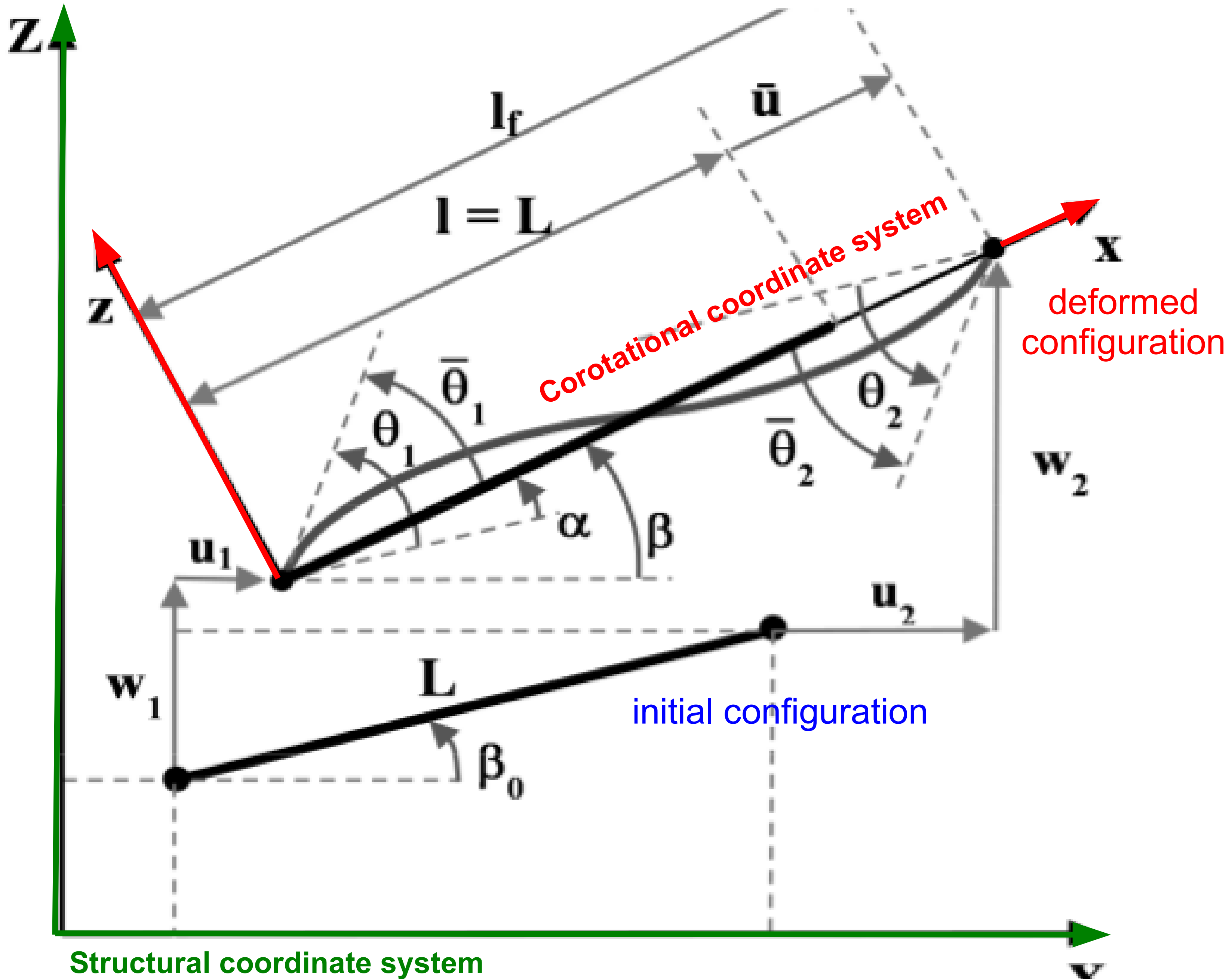
Corotational Bernoulli beam formulation



Global to local displacements

$$\mathbf{q}_{global}^{T} = \left\{ u_1 \quad w_1 \quad \theta_1 \quad u_2 \quad w_2 \quad \theta_2 \right\}$$

$$\begin{bmatrix} \bar{u} \\ \bar{\theta}_1 \\ \bar{\theta}_2 \end{bmatrix} = \begin{bmatrix} l_f - L \\ \theta_1 - \alpha \\ \theta_2 - \alpha \end{bmatrix} = \begin{bmatrix} l_f - l \\ \theta_1 - \beta - \beta_0 \\ \theta_2 - \beta - \beta_0 \end{bmatrix}$$

rigid rotation

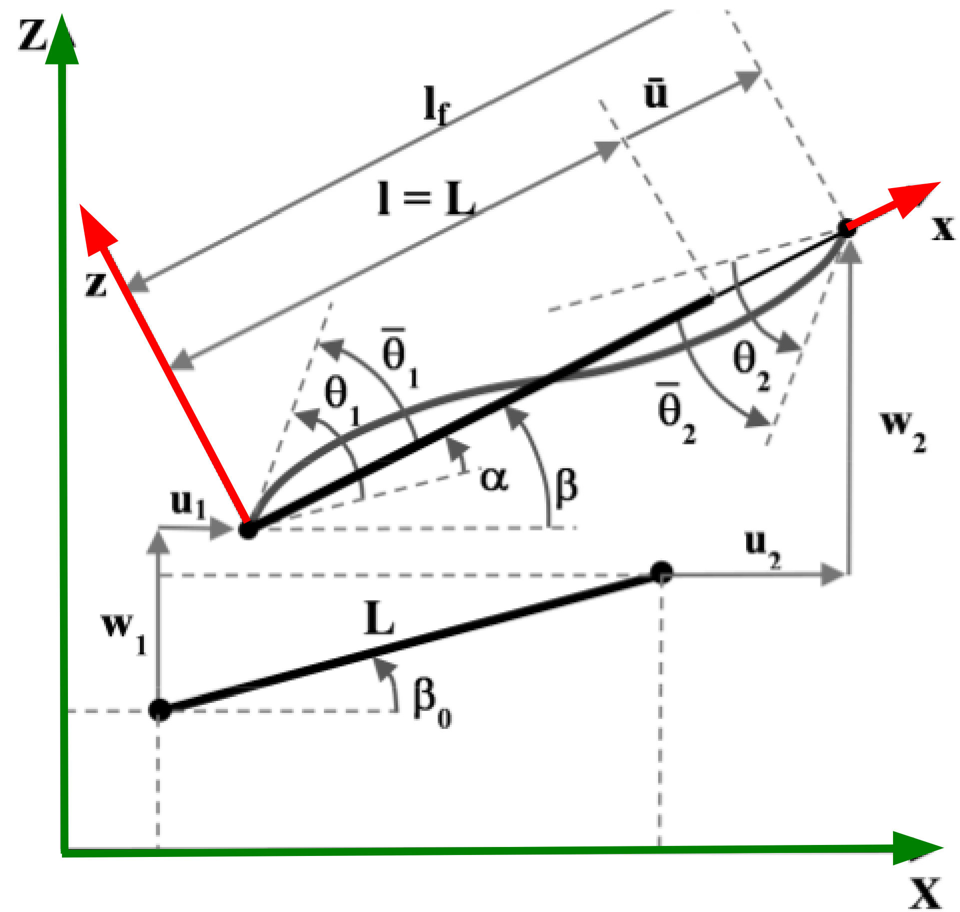
$$\mathbf{q}_{local}{}^{T} = \left\{ \bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2 \right\}$$

$$c_o = \cos\ \beta_o = \frac{1}{L}(X_2 - X_1)$$

$$s_o = \sin\ \beta_o = \frac{1}{L}(Z_2 - Z_1)$$

$$c = \cos\ \beta = \frac{1}{l_f}(X_2 - X_1 + u_2 - u_1)$$

$$s = \sin\ \beta = \frac{1}{l_f}(Z_2 - Z_1 + w_2 - w_1)$$



$$\sin\alpha = c_o s - s_o c$$

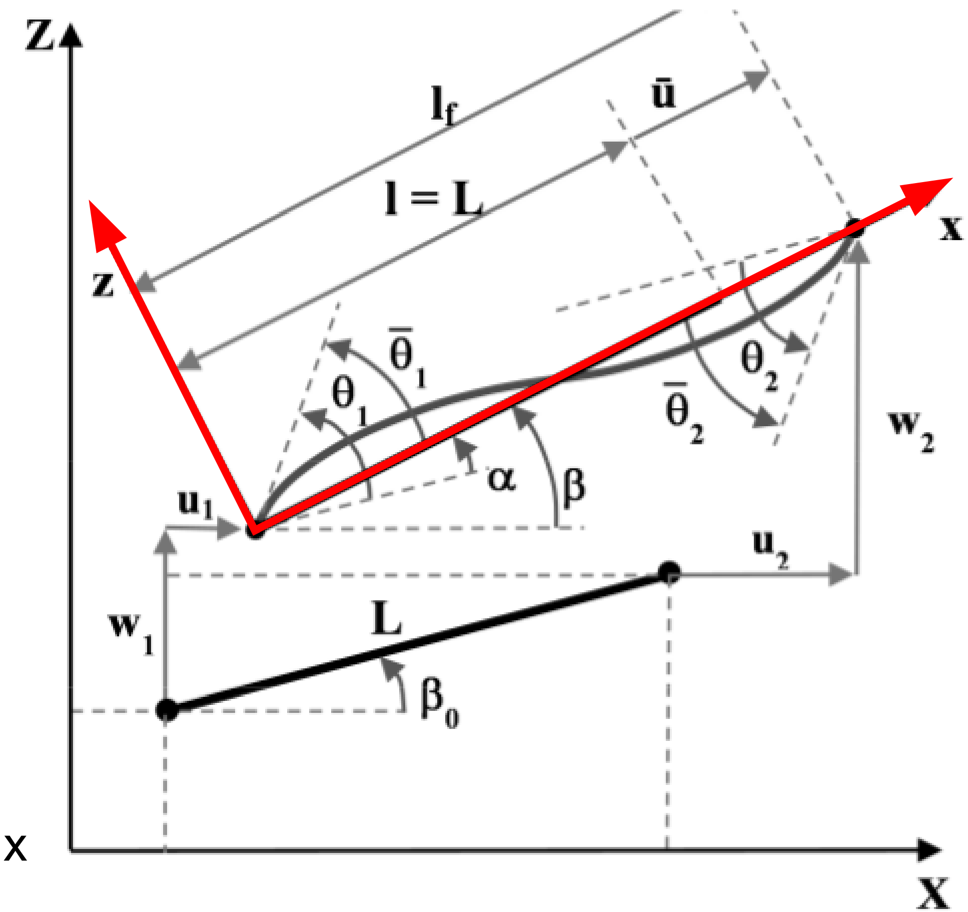
$$\cos\alpha = c_o c - s_o s$$

rigid rotation

Strains and stresses in corotational system

$$\mathbf{q}_{local}{}^{T} = \left\{ \bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2 \right\}$$

naturally decoupled from rigid rotation



Interpolation of local displacements

$$u_{local} = \frac{x}{l}\bar{u}$$

$$w_{local} = x\left(1 - \frac{x}{l}\right)^2 \bar{\theta}_1 + \frac{x^2}{l}\left(\frac{x}{l} - 1\right)\bar{\theta}_2$$

conjugated

► Generalized strains

$$\begin{bmatrix} \bar{\varepsilon} \\ \chi \end{bmatrix} = \begin{bmatrix} \frac{\partial u_{local}}{\partial x} \\ \frac{\partial^2 w_{local}}{\partial x^2} \end{bmatrix}$$

► Generalized stresses

$$N = \int_{a_{cor}} \sigma \, da$$

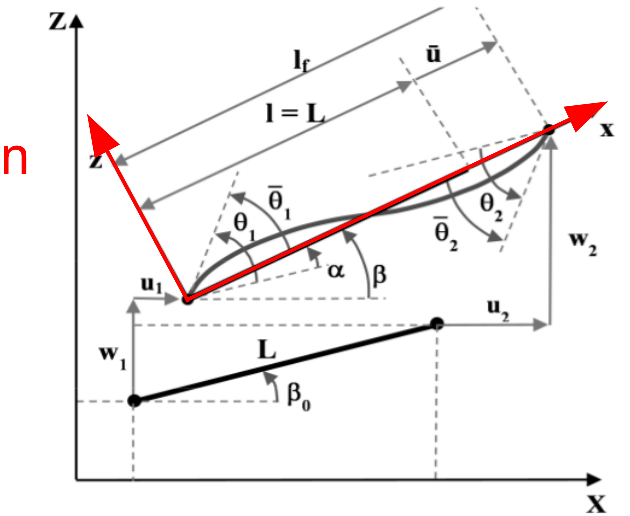
$$M = -\int_{a_{cor}} \sigma z \, da$$

vary along x

Strains and stresses in corotational system

$$\mathbf{q}_{local}{}^{T} = \left\{ \bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2 \right\}$$

naturally decoupled from rigid rotation



Interpolation of local displacements

$$u_{local} = \frac{x}{l}\bar{u}$$

$$w_{local} = x\left(1 - \frac{x}{l}\right)^2 \bar{\theta}_1 + \frac{x^2}{l}\left(\frac{x}{l} - 1\right)\bar{\theta}_2$$

Axial strain at a given beam depth (Bernoulli)

$$\varepsilon(x,z) = \frac{\partial u_{local}}{\partial x} - \chi z = \frac{\bar{u}}{l} - z\left[\left(6\frac{x}{l^2} - \frac{4}{l}\right)\bar{\theta}_1 + \left(6\frac{x}{l^2} - \frac{2}{l}\right)\bar{\theta}_2\right]$$

Internal work

$$W_{\text{int}} = \int_{v_{cor}} \sigma\, \delta\varepsilon\, dv$$

$$W_{\text{int}} = \int_{v_{cor}} \sigma \left\{ \frac{\delta\bar{u}}{l} - z\left[\left(6\frac{x}{l^2} - \frac{4}{l}\right)\delta\bar{\theta}_1 + \left(6\frac{x}{l^2} - \frac{2}{l}\right)\delta\bar{\theta}_2\right]\right\} dv$$

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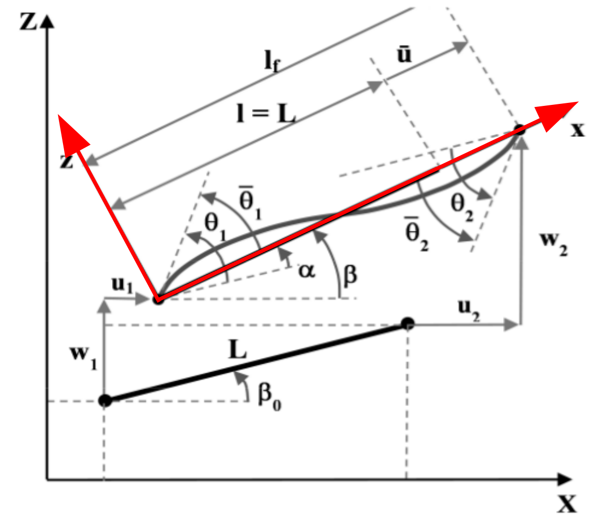
Strains and stresses in corotational system

$$\mathbf{q}_{local}{}^{T} = \left\{ \bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2 \right\}$$

naturally decoupled from rigid rotation



conjugated

Internal work

$$W_{\text{int}} = \int_{v_{cor}} \sigma \, \delta\varepsilon \, dv$$

$$W_{\text{int}} = \int_{v_{cor}} \sigma \left\{ \frac{\delta \bar{u}}{l} - z \left[\left(6\frac{x}{l^2} - \frac{4}{l} \right) \delta\bar{\theta}_1 + \left(6\frac{x}{l^2} - \frac{2}{l} \right) \delta\bar{\theta}_2 \right] \right\} dv$$

Internal forces

$$\bar{N} = \int_{v_{cor}} \frac{\sigma}{l} \, dv = \int_{l} \frac{N}{l} \, dl$$

$$\bar{M}_1 = -\int_{v_{cor}} \sigma z \left(6\frac{x}{l^2} - \frac{4}{l} \right) dv = \int_{l} M \left(6\frac{x}{l^2} - \frac{4}{l} \right) dl$$

$$\bar{M}_2 = -\int_{v_{cor}} \sigma z \left(6\frac{x}{l^2} - \frac{2}{l} \right) dv = \int_{l} M \left(6\frac{x}{l^2} - \frac{2}{l} \right) dl$$

with

$$\begin{cases} N = \displaystyle\int_{a_{cor}} \sigma \, da \\ M = -\displaystyle\int_{a_{cor}} \sigma z \, da \end{cases}$$

evaluated at Gauss points

Local to global forces

conjugated

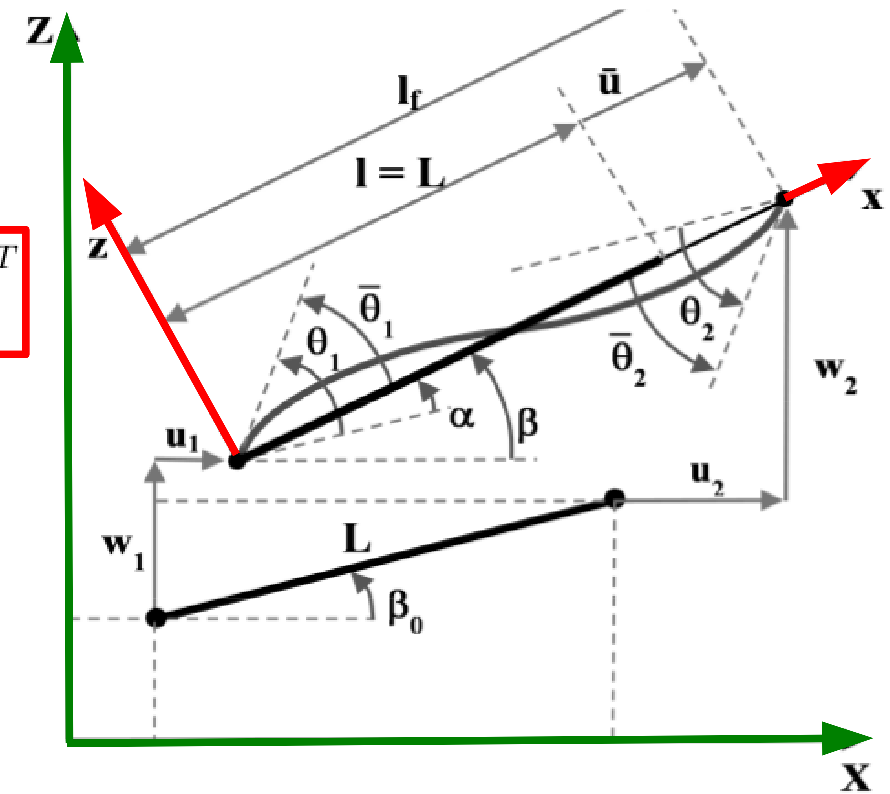
$$\mathbf{q}_{global}^T = \left\{ u_1 \quad w_1 \quad \theta_1 \quad u_2 \quad w_2 \quad \theta_2 \right\}$$

$$\left\{ f_1^x \quad f_1^y \quad c_1 \quad f_2^x \quad f_2^y \quad c_2 \right\}^T = \mathbf{T}^T \left\{ \bar{N} \quad \bar{M}_1 \quad \bar{M}_2 \right\}^T$$

with $\mathbf{T} = \begin{bmatrix} 1 & 1/l_f & 1/l_f \end{bmatrix} \begin{bmatrix} -c & -s & 0 & c & s & 0 \\ s & -c & 1 & s & c & 0 \\ s & -c & 0 & s & c & 1 \end{bmatrix}$

$$c = \cos\ \beta = \frac{1}{l_f}(X_2 - X_1 + u_2 - u_1)$$

$$s = \sin\ \beta = \frac{1}{l_f}(Z_2 - Z_1 + w_2 - w_1)$$



Computation of internal forces - summary

$$\mathbf{q}_{global}^T = \{u_1 \quad w_1 \quad \theta_1 \quad u_2 \quad w_2 \quad \theta_2\}$$

naturally decoupled from rigid rotation

$$\mathbf{q}_{local}{}^T = \{\bar{u} \quad \bar{\theta}_1 \quad \bar{\theta}_2\}$$

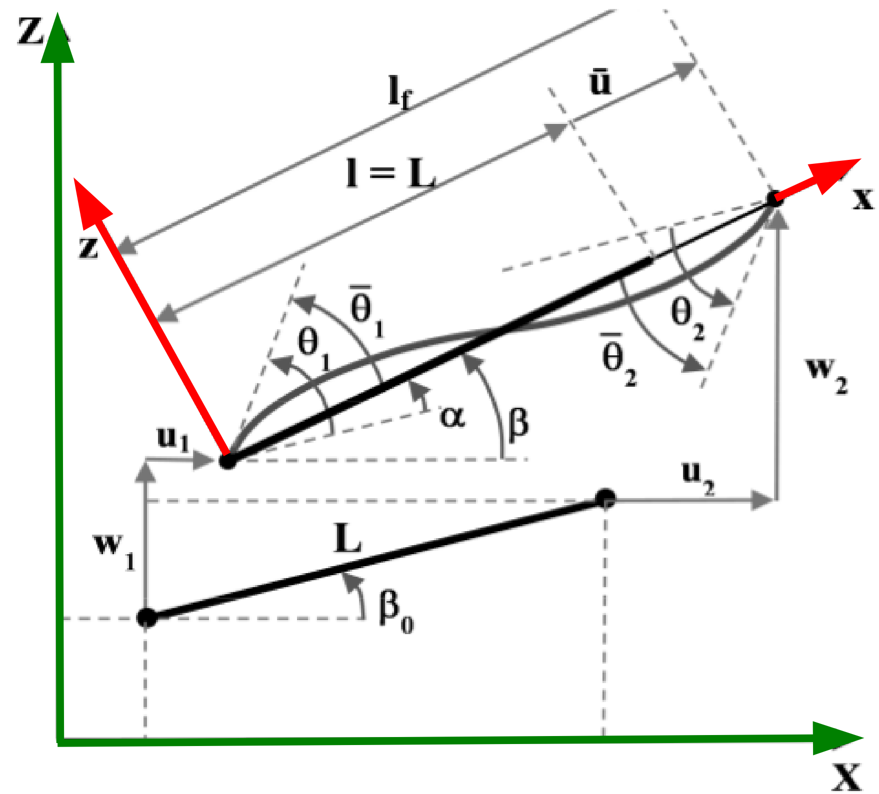
conjugated

$$\left.\begin{matrix} u_{local} \\ w_{local} \end{matrix}\right\} \rightarrow \begin{bmatrix} \bar{\varepsilon} \\ \chi \end{bmatrix} \xrightarrow{\text{conjugated}} \begin{bmatrix} N \\ M \end{bmatrix}$$

Gauss pt level

conjugated

$$\{f_1^x \quad f_1^y \quad c_1 \quad f_2^x \quad f_2^y \quad c_2\}^T = \mathbf{T}^T \{\bar{N} \quad \bar{M}_1 \quad \bar{M}_2\}^T$$



Structural stiffness matrix

$$\delta \mathbf{f}_{global,\text{int}} = \delta\left(\mathbf{T}^T \mathbf{f}_{local,\text{int}}\right)$$

with $\mathbf{T} = \begin{bmatrix} 1 & 1/l_f & 1/l_f \end{bmatrix} \begin{bmatrix} -c & -s & 0 & c & s & 0 \\ s & -c & 1 & s & c & 0 \\ s & -c & 0 & s & c & 1 \end{bmatrix}$

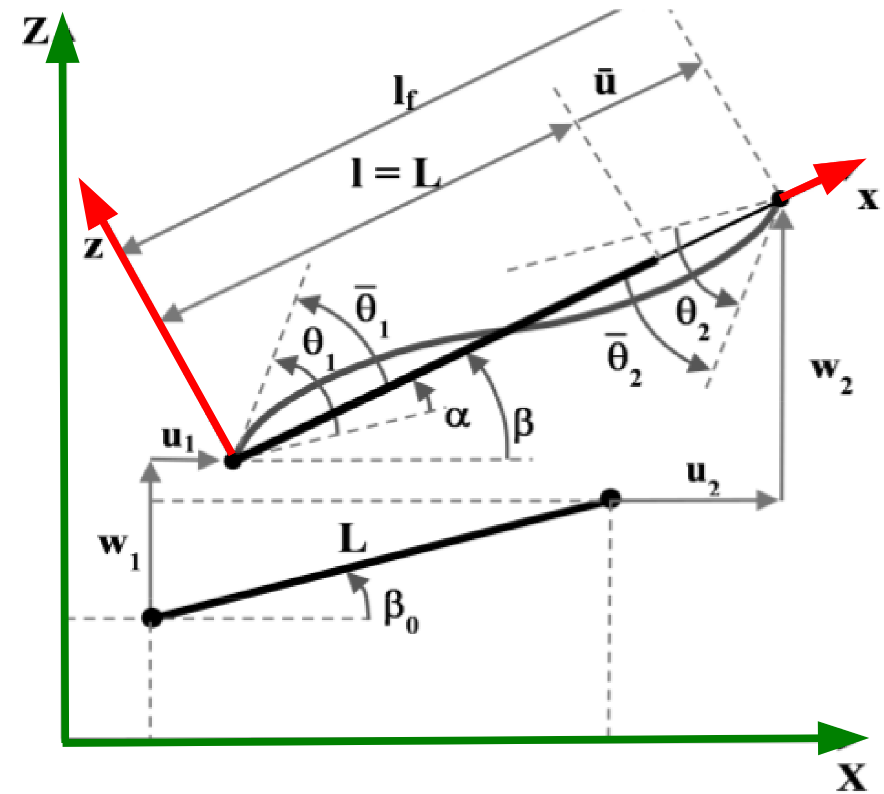
$$\delta \mathbf{f}_{global,\text{int}} = \mathbf{T}^T \delta \mathbf{f}_{local,\text{int}} + \bar{N}\delta \mathbf{t}_1 + \bar{M}_1 \delta \mathbf{t}_2 + \bar{M}_2 \delta \mathbf{t}_3$$

$$\mathbf{T}^T \delta \mathbf{f}_{local,\text{int}} = \mathbf{T}^T \mathbf{K}_{local} \delta \mathbf{q}_{local} = \mathbf{T}^T \mathbf{K}_{local} (\mathbf{T} \delta \mathbf{q}_{global})$$

$$\mathbf{K}_{local} = \int_l \mathbf{B}^T \mathbf{H} \mathbf{B}\, dl$$

sectional stiffness

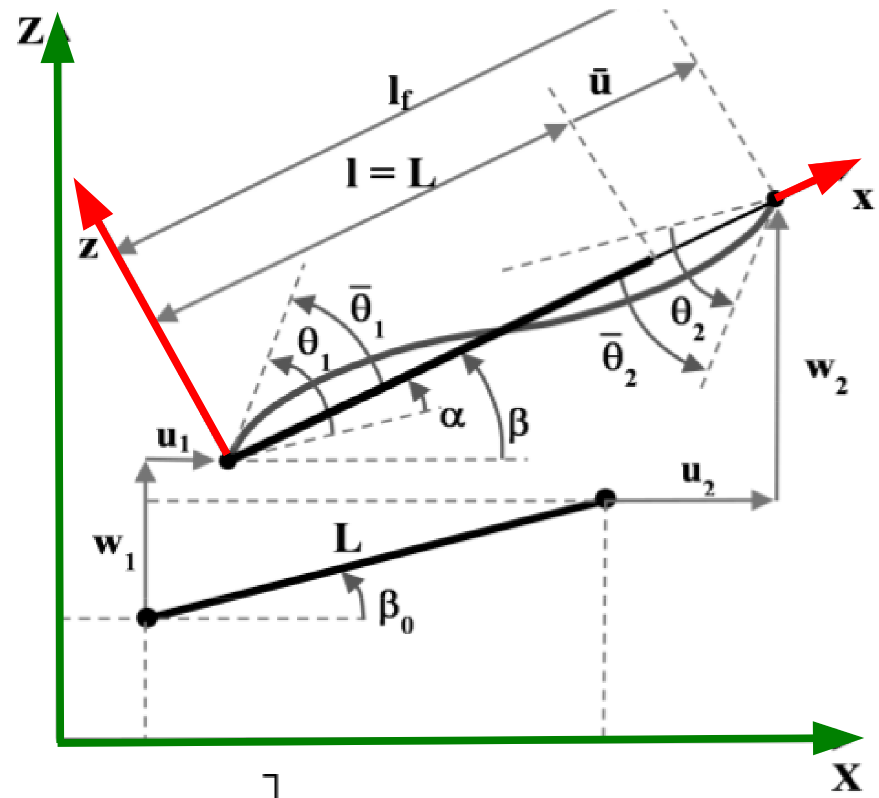
$$H = \frac{dE_{gen}}{d\Sigma_{gen}} \quad \text{with} \quad E_{gen} = \begin{bmatrix} \bar{\varepsilon} \\ \chi \end{bmatrix} \quad \Sigma_{gen} = \begin{bmatrix} N \\ M \end{bmatrix}$$



Structural stiffness matrix

$$\delta \mathbf{f}_{global,\text{int}} = \delta\left(\mathbf{T}^T \mathbf{f}_{local,\text{int}}\right)$$

with $\mathbf{T} = \begin{bmatrix} 1 & 1/l_f & 1/l_f \end{bmatrix} \begin{bmatrix} -c & -s & 0 & c & s & 0 \\ s & -c & 1 & s & c & 0 \\ s & -c & 0 & s & c & 1 \end{bmatrix}$



$$\delta \mathbf{f}_{global,\text{int}} = \mathbf{T}^T \delta \mathbf{f}_{local,\text{int}} + \bar{N}\delta \mathbf{t}_1 + \bar{M}_1 \delta \mathbf{t}_2 + \bar{M}_2 \delta \mathbf{t}_3$$

$$\mathbf{T}^T \delta \mathbf{f}_{local,\text{int}} = \mathbf{T}^T \mathbf{K}_{local} \delta \mathbf{q}_{local} = \mathbf{T}^T \mathbf{K}_{local} (\mathbf{T} \delta \mathbf{q}_{global})$$

$$\delta \mathbf{f}_{global,\text{int}} = \left[\mathbf{T}^T \mathbf{K}_{local}\, \mathbf{T} + \bar{N}\frac{\mathbf{z}\mathbf{z}^T}{l_f} + (\bar{M}_1 + \bar{M}_2)\frac{1}{l_f^2}\left(\mathbf{r}\mathbf{z}^T + \mathbf{z}\mathbf{r}^T\right)\right]\delta \mathbf{q}_{global}$$

$$\mathbf{K}_{global} = \mathbf{T}^T \mathbf{K}_{local} \mathbf{T} + \bar{N}\frac{\mathbf{z}\mathbf{z}^T}{l_f} + (\bar{M}_1 + \bar{M}_2)\frac{1}{l_f^2}\left(\mathbf{r}\mathbf{z}^T + \mathbf{z}\mathbf{r}^T\right)$$

Contribution of the stress variation

Other terms: contribution of the change in the geometry

$$c = \cos\ \beta = \frac{1}{l_f}(X_2 - X_1 + u_2 - u_1)$$

$$s = \sin\ \beta = \frac{1}{l_f}(Z_2 - Z_1 + w_2 - w_1)$$

Recommended literature

J.-M. Battini, Corotational beam elements in instability problems, *Ph.D. Thesis*, Royal Institute of Technology, Department of Mechanics, Stockholm, Sweden, 2002.

C.E.M. Oliveira, A.G. Marchis, P.Z. Berke, R.A.M. Silveira, T.J. Massart, Computational analysis of a RC planar frame using corotational multilayered beam FE, correlated to experimental results, In Proceedings of XXXIV CILAMCE, 13 pages, 2013